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## NOTICE.

It has been found necessary, in the present Rudimentary Treatise on the interesting subject of Lighthouses-in order to the due development of its elementary and practical character, and to ensure its utility to students and the practical engineer, and its comprehensive elucidation by the author, for the use of those not professionally engaged-to extend it to a treble part. This unavoidable increase of its bulk is the only excuse offered for the price being made $3 s$.; and it is anticipated that, when the matter is investigated,-taking into account that there are 14 engravings, 104 woodcuts, and 15 sheets, of 24 pages each, of expensively-printed matter, the publisher will receive the approbation rather than the censure of the public.
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# RUDIMENTARY TREATISE 

ON THE

# HISTORY, CONSTRUCTION, and ILLUMINATION 

OF
LIGHTHOUSES.

BY
ALAN Stevenson, LL.B., F.R.S.E., M.I.C.E., ENGINEER TO THE BOARD OF NORTHEMN LIGATHOUSES.

OAth \#IIustratibe Engrabings and ©

LONDON:
JOHN WEALE, 59 HIGH HOLBORN.
1850.

The following pages contain my Notcs on the History and the Illumination of Lighthouses, with a few additions in various places, and Excerpts from my Account of the Skerryvore Lighthouse, which were originally prepared at the desire of the Commissioners of the Northern Lighthouses. I glally embrace this opportunity of aeknowledging the liberality of the Comaissioners, who have now put them at ny disposal for reproduction in any form that might seem desirable.
A. S.

EDinburgh, December 30, 1849.

# MONSIEUR LÉONOR FRESNEL, 

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insPECTEGA DIVISIONNAIRE DES PONTS ET CHAUSEEES, hecrétaire émérite de la commission des piares,
``` \&c. \&c. \&c.

My Dear Mr. Fresnel,
There is no one to whom I can dedicate this small Volume so properly as to you. Much of what it contains is founded on information which I owe to your generosity and friendship. A great part of it, also, is devoted to a description of the beautiful system of Lighthouse Illumination invented by your late distinguished Brother, Augustin Fresnel; who, to the high intellectual endowments which have extended his fame over the whole scientific world, united, in a remarkable degree, those amiable qualities which endeared him much to all who knew him, and most to those who knew him best; and from whom, when, in early youth,

I accompanied my Father to Paris, I experienced much cousideration and kindness :

Multis ille bonis flebilis occidit;
Nulli flebilior quam tibi :
I am,

> My dear Friend,

With every sentiment of affection and respect,
Very faithfully yours,

\author{
Alan Stevenson.
}

Edinhurgh. Dec. 21, 1849.

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\section*{TREATISE ON LIGHTHOUSES.}

\section*{HISTORY OF LIGHTHOUSES, ANCIENT AND MODERN.}

The early history of Lighthouses is very uncertain; and some ingenious antiquaries, finding the want of authentic records, have been anxious to supply the deficiency by conjectures based upon casual and obscure allusions in ancient writers, and by vague hypotheses drawn from the heathen mythology. Certain writers have gone so far as to imagine that the Cyclopes were the keepers of lighthouses; whilst others have actually maintained that Cyclops was intended, by a bold prosopopœia, to represent a lighthouse itself! A notion so fanciful deserves little consideration; and very ill accords with that mythology of which it is intended to be an exposition, as seems sufficiently plain from a passage in the ninth Odyssey (l. 146), where Homer (who flourished about 907 B.C.), after describing the darkness of the night, in-
forms us that the fleet of Ulysses actually struck the shore of the Cyclopean island, before it could be seen. On the faith, also, of similar obscure and finely-drawn etymologies, various places, such as Calpe and Abyla, the opposite points of Africa and Europe, at the Straits of the Mediterranean, have been unhesitatingly recognised as the sites of celebrated light-towers; and the Latin words turris and columna have been supposed primarily to signify a lighthouse, the first being written Tor-is, the Tower of fire, and the second Col-on, the Pillar of the Sun.

Nor does there appear any better reason for supposing, that, under the history of Tithonus, Chiron, or any other personage of antiquity, the idea of a lighthouse was conveyed; for such suppositions, however reconcileable they may appear with some parts of mythology, involve obvious inconsistencies with others. It seems, indeed, most improbable, that, in those early times, when navigation was so little practised, the advantages of bea-con-lights were so generally known and acknowledged, as to render them the objects of mythological allegory.

It must not, however, be imagined, that ancient writings are entirely destitute of allusions to the subject of beacon-lights for the guidance of the mariner. The venerable poet already noticed, in speaking of the shield of Achilles, has beautifully described the flash of a bea-con-light in some solitary place, as seen by seamen leaving their friends, in lines, which contain ample proof of the existence of such a provision for the safety of the mariner in Homer's time.-Il., xix. 375.

In the English Bible the word beacon occurs but once, and that in the Prophecies of Isaiah (xxx. 17), who
lived above 200 years later than Homer ; but the original, as translated by the Seventy, merely imports a flagstaff or perch, and does not at all imply the knowledge of beacon-lights among the Hebrews, who were not a maritime people.

About 300 years before the Christian era, Chares, the disciple of Lysippus, constructed the celebrated brazen statue, called the Colossus of Rhodes. It was of such dimensions as to allow vessels to sail into the harbour between its legs, which spanned the entrance. There is considerable probability in the idea, that this figure served the purposes of a lighthouse; but there is no passage in any ancient writer, where this use of the Colossus is expressly mentioned. Many inconsistencies occur in the account of this fabric by early writers, who, in describing the distant objects which could be seen from it, appear to have forgotten the corresponding height which they must thus assign to the figure. The statue was partly demolished by an earthquake, about eighty years after its completion; and so late as the year 672 of our era the brass of which it was composed was sold by the Saracens to a Jewish merchant of Edessa, for a sum, it is said, equal to \(£ 36,000\).

Little is known with certainty regarding the Pharos of Alexandria, which was regarded by the ancients as one of the seven wonders of the world. It was built in the reign of Ptolemy Philadelphus, about 300 years before the Christian era; and Strabo relates that Sostratus, a friend of the royal family, was the architect. He describes it as built in a wonderful manner, in many storeys of white stone, on a rock forming the promon-
tory of the island Pharos (whence the tower derived its name), and says that the building bore the inscription : "Sostratus of Cnidos, the son of Dexiphanes, to the Gods, the Saviours, for the benefit of seamen." He concludes his brief notice of it by describing the neighbouring shores as low and encumbered with shoals and snares, and as calling for the establishment of a lofty and bright beacon, as a sign for sailors arriving from the ocean to guide them into the entrance to the haven.(See Sträbo, Oxon., 1807, page 1123; Plin. Nat. Hist., ii. 87 , v. 31, xiii. 21.)

The accounts which have come down to us of the dimensions of this remarkable edifice are exceedingly various; and the statements of the distance at which it could be seen are clearly fabulous. That of Josephus (who likens it to the second of Herod's three towers at Jerusalem, called Phasael, in honour of his brother) is the least removed from probability; yet even he informs us, that the fire which burnt on the top to enable seamen to anchor in sight of it, before coming near the shore, and so to avoid the difficulty of the navigation by night, was visible at a distance equal to about thirtyfour English miles. Such a range for a lighthouse on the low shores of Egypt, would require a tower about 550 feet in height!* Ammianus Marcellinus \(\dagger\) and Pliny \(\ddagger\) are both very circumstantial in their notices of the Pharos as a beacon-light to guide seamen in ap-

\footnotetext{
* Bell. Judaic. iv., cap. 10, sec. 5, and vi., sec. 3.
\(\dagger\) Ammianus Marcellinus, l. xxii., c. 16.
\(\ddagger\) Plinii Hist. Nat., xxxvi. 18.
}
proaching the coast of Egypt and port of Alexandria. The latter adds the interesting fact, that the cost of the tower was reckoned at a sum equal to about \(£ 390,000\) of our money; and both of them agree in stating, that a light was shewn from it at night. Ammianus Marcellinus differs from all the other writers, in attributing the erection of the tower to Queen Cleopatra. Pliny mentions in passing, that there were also lighthouses at Ostia and Ravenna.

If the reports of some writers are to be believed, this tower must have far exceeded in size the great Pyramid itself; but the fact that a building, of comparatively so late a date, should have so completely disappeared, whilst the Pyramid remains almost unchanged, is a sufficient reason for rejecting, as erroneous, the dimensions which have been assigned by most writers to the Pharos of Alexandria. Some have pretended that large mirrors were employed to direct the rays of the beacon-light on its top, in the most advantageous direction; but, in so far as I know, there is no definite evidence in favour of this supposition. Others, with greater probability, have imagined that this celebrated beacon was known to mariners, simply by the uncertain and rude light afforded by a common fire. The poet Lucan, on most occasions sufficiently fond of the marvellous, speaks of the Pharos as having indicated to Julius Cæsar his approach to Egypt on the seventh night after he sailed from Troy; but he takes no notice of the gigantic mirrors which it is said to have contained. It is true that, by using the word " lampada," which can only with propriety be applied to a more perfect mode of illumination than an
open fire, he appears to indicate that the "fammis" of which he speaks, were not so produced. The word "lampada" may, however, be used metaphorically, and "flammis" would, in this case, notimproperly describe the irregular appearance of a common fire.*

Perhaps, also, the opinion that some kind of lamp was used in the Pharos, may seem to receive countenance from the remarkable words of Pliny, in the passage above cited-" Periculum in continuatione ignium, ne sidus existimetur, quoniam è longinquo similis flammarum aspectus est." The fear he expresses. lest the light viewed from a distance should be mistaken for a star, could hardly be applicable to the diffuse, oscillating, lambent light derived from an open fire, and certainly gives some reason for imagining that, even at that remote time, the art of illuminating lighthouses was better understood than in the early part of the present century. Casual notices of the Pharos are also to be found in Cæsar's Commentaries, Valerius Flaccus, and Pomponius Mela. \(\dagger\) At Alexandria there is a modern lighthouse called the Pharos, which is maintained by the Pacha of Egypt. It may not, perhaps, be unwarrantable to suppose that the word Pharos, as applied to a lighthouse, is a Hellenic form of Phrah, the Egyptian name of the Sun.

Mr Moore, in his History of Ireland, vol. i., p. 16, speaks of the Tower of Coruña, which he says is mentioned in the traditionary history of that country as a lighthouse erected for the use of the Irish in their fre-

\footnotetext{
* Pharsal., ix. 1004.
\(\dagger\) Cæsar de Bell. Civil., iii; Argonaut, vii., 84 ; Pompon. Mela, ii., 7.
}
quent early intercourse with Spain. In confirmation of this opinion, he cites a somewhat obscure passage from厌thicus, the cosmographer. This, in all probability, is the tower which Humboldt mentions in his Narrative under the name of the Iron Tower, which was built as a lighthouse by Caius Saevius Lupus, an architect of the city of Aqua Flavia, the modern Chaves. A lighthouse has lately been established on this headland, for which dioptric apparatus was supplied from the workshop of M. Létourneau of Paris.*

There is also a record in Strabo of a magnificent lighthouse of stone at Capio, or Apio, near the harbour of Menestheus (the modern Mesa Asta or Puerto de Sta. Maria), built on a rock nearly surrounded by the sea, as a guide for the shallows at the mouth of the Guadalquiver, which he describes in terms almost identical with those used by him in speaking of the Pharos of Alexandria. I am not aware of any other notice of this great work, for such it seems to have been, to have deserved the praises of Strabo. \(\dagger\)

In Camden's Britannia, a passing notice is taken of the ruins called Cobar's Altar, at Dover, and of the Tour d'Ordre, at Boulogne, on the opposite coast; both of which are conjectured, on I know not what authority, to have been ancient lighthouses. Pennant describes the remains of a Roman Pharos near Holywell, but cites no authorities for his opinion as to its use. There were likewise remains of a similar structure at Flamboroughhead.

\footnotetext{
* See also a curious account of the traditions about this tower in Southey's Letters from Spain and Portugal, p. 17.
\(\dagger\) Oxon. 1807, p. 184.
}

A very meagre and unintelligible account is also given of a lighthouse at St Edmund's Chapel, on the coast of Norfolk, in Gough's additions to Camden, by which it might seem that the lighthouse was erected in 1272.*

Such seems to be the sum of our knowledge of the ancient history of lighthouses, which, it must be admitted, is neither accurate nor extensive. Our information regarding modern lighthouses is of course more minute in its details and more worthy of credit. The greater part of it is drawn from authentic sources; and much of what is afterwards stated is the result of my own observation, during my visits to the most important lighthouses of Europe.
The first lighthouse of modern days that merits attention, is the Tour de Corduan, which, in point of architectural grandeur, is unquestionably the noblest edifice of the kind in the world. It is situate on an extensive reef at the mouth of the river Garonne, and serves as a guide to the shipping of Bourdeaux and the Languedoc Canal, and indeed of all that part of the Bay of Biscay. It was founded in the year 1584, but was not completed till 1610, under Henri IV. It is minutely described in Belidor's Architecture Hydraulique. The building is 197 feet in height, and consists of a pile of masonry, forming successive galleries, enriched with pilasters and friezes, and rising above each other with gradually

\footnotetext{
* Gongh's Camden's Britannia, vol. i., 318, and vol. ii., p. 198. Batcheller, in his Dover Guide (1845, p. 111), says, that the Dover Pharos was built "during the lieutenancy of Aulus Plautius and Ostorius Scapula, the latter of whom left Britain A.D. 53." -Pennant's History of Whiteford and Holywell, p. 112.
}
diminished diameters. Those galleries are surmounted by a conical tower, which terminates in the lantern. Round the base is a wall of circumvallation, 134 feet in diameter, in which the lightkeepers' apartments are formed, somewhat in the style of casemates. This wall is an outwork of defence, and receives the chief shock of the waves. The tower itself contains a chapel, and various apartments; and the ascent is by a spacious staircase. The first light exhibited in the Tour de Corduan was obtained by burning billets of oak wood, in a chauffer at the top of the tower; and the use of coal instead of wood, was the first improvement which the light received. A rude reflector, in the form of an inverted cone, was afterwards added, to prevent the loss of light which escaped upwards. About the year 1780, M. Lenoir was employed to substitute paraboloidal reflectors and lamps; and in 1822, the light received its last improvement, by the introduction of the dioptric instruments of Augustin Fresnel, the celebrated French Academician.

The history of the famous lighthouse on the Eddystone Rocks is well known to the general reader, from the " Narrative" of Smeaton the Engineer. Those rocks are \(9 \frac{1}{2}\) miles distant from the Ram-Head, on the coast of Cornwall; and from the small extent of the surface of the chief rock and its exposed situation, the construction of the lighthouse was a work of very great difficulty: The first erection was of timber, designed by Mr Winstanley; and was commenced in 1696. The light was exhibited in November 1698. It was soon found, however, that the sea rose upon that tower to a much greater height than had been anticipated; so much so,
it is said, as to "bury under the water" the lantern, which was sixty feet above the rock; and the engineer was therefore afterwards under the necessity of enlarging the tower, and carrying it to the height of 120 feet. In November 1703, some considerable repairs were required, and Mr Winstanley, accompanied by his workmen, went to the lighthouse to attend to their execution; but the storm of the 26th of that month carried away the whole erection, when the engineer and all his assistants unhappily perished!

The want of a light on the Eddystone, soon led to a fatal accident; for not long after the destruction of Mr Winstanley's lighthouse, the Winchilsea man-of-war was wrecked on the Eddystone Rocks, and most of her crew were lost. Three years, however, elapsed, after this melancholy proof of the necessity for a light, before the Trinity-House of London could obtain a new Act of Parliament, to extend their powers; and it was not till the month of July 1706, that the construction of a new lighthouse was begun under the direction of Mr John Rudyerd of London. On the 28th of July 1708, the new light was first shewn, and it continued to be regularly exhibited till the year 1755, when the whole fabric was destroyed by accidental fire, after it had stood fortyseven years. But for this circumstance, it is impossible to tell how long the lighthouse might, with occasional repair, have lasted, as Mr Rudyerd seems to have executed his task with much judgment, carefully rejecting all architectural decoration, as unsuitable for such a situation, and directing his attention to the formation of a tower which should offer the least resistance to the
waves. The height of the tower, which was of a conical form, and constructed of timber, was 92 feet, including the lantern; and the diameter at the base, which was a little above the level of high water, was 23 feet.

The advantages of a light on the Eddystone having been so long known and acknowledged by seamen, no time was permitted to elapse before active measures were taken for its restoration; and Smeaton, to whom application was made for advice on the subject, recommended the exclusive use of stone as the material, which, both from its weight and other qualities, he considered most suitable for the situation. On the 5th of April 1756, Smeaton first landed on the rock, and made arrangements for erecting a lighthouse of stone and preparing the foundations, by cutting the surface of the rock into regular horizontal benches, into which the stones were carefully dovetailed or notched. The first stone was laid on 12th June 1757, and the last on the 24th of August 1759. The tower measures 68 feet in height, and 26 feet in diameter at the level of the first entire course; and the diameter under the cornice is 15 feet. The first 12 feet of the tower form a solid mass of masonry; and the stones of which it is composed are united by means of stone joggles, dovetailed joints, and oaken treenails. It is remarkable that Smeaton should have adopted an arched form for the floors of his building, instead of employing the floors as tie-walls formed of dovetailed stones. To counteract the injurious tendency of the outward thrust of those arched floors, he had recourse to the ingenious expedient of laying, in circular trenches or grooves cut in
the stones which form the outside casing, tie-belts of chain which were heated before being set in the grooves by means of an application of hot lead, and became tight on cooling after they were fixed in the wall. The light was exhibited on the 16 th October 1759 ; but such was the state of lighthouse apparatus in Britain at that period, that a feeble light from tallow candles was all that decorated this noble structure. In 1807, when the property of this lighthouse again came into the hands of the Trinity-House, at the expiry of a long lease, Argand burners, and paraboloïdal reflectors of silvered copper, were substituted for the chandelier of candles.

The dangerous reef called the Inch Cape or Bell Rock, so long a terror to mariners, was well known to the earliest navigators of Scotland. Its dangers were so generally acknowledged, that the Abbots of Aberbrothwick, from which the rock is distant about twelve miles, caused a float to be fixed upon the rock with a bell attached to it, which, being swung by the motion of the waves, served by its tolling to warn the mariner of his approach to the reef. From this circumstance, which formed the groundwork of Southey's striking ballad of Sir Ralph the Rover, the rock is said to have derived its name. Amongst the many losses which occurred on the Bell Rock in modern times, one of the most remarkable is that of the York, seventy-four, with all her crew, part of the wreck having been afterwards found on the rock, and part having come ashore on the neighbouring coast. During the survey of the rock, also, several instances were discovered of the extent of loss which this reef had occasioned; and many articles
of ship's furnishings were picked up on it, as well as various coins, a bayonet, a silver shoe-buckle, and many other small objects. Impressed with the great importance of some guide for the Bell Rock, Captain Brodie, R.N., set on foot a small subscription, and erected a beacon of spars on the rock, which, however, was soon destroyed by the sea. He afterwards constructed a second beacon, which soon shared the same fate. It was not, therefore, until 1802, when the Commissioners of Northern Lighthouses brought a bill into Parliament for power to erect a lighthouse on it, that any efficient measures were contemplated for the protection of seamen from this rock, which, being covered at every springtide to the depth of from twelve to sixteen feet, and lying right in the fairway to the Friths of Forth and Tay, had been the occasion of much loss both of property and life. In 1806, the bill passed into a law; and various ingenious plans were suggested for overcoming the difficulties which were apprehended, in erecting a lighthouse on a rock twelve miles from land, and covered to the depth of twelve feet by the tide. But the suggestion of Mr Robert Stevenson, the engineer to the Lighthouse Board, after being submitted to the late Mr Rennie, was at length adopted; and it was determined to construct a tower of masonry, on the principle of the Eddystone. On the 17th of August 1807, Mr Stevenson accordingly landed with his workmen, and commenced the work by preparing the rock to receive the supports of a temporary pyramid of timber, on which a barrack-house for the reception of the workmen was to be placed; and during this operation, much hazard was
often incurred in transporting the men from the rock, which was only dry for a few hours at spring-tides, to the vessel which lay moored off it. The lowest floor of this temporary erection, in which the mortar for the building was prepared, was often broken up and removed by the force of the sea. The foundation for the tower having been excavated, the first stone was laid on the 10th July 1808, at the depth of sixteen feet below the high water of spring-tides; and at the end of the second season, the building was five feet six inches above the lowest part of the foundation. The third season's operations terminated by finishing the solid part of the structure, which is thirty feet in height; and the whole of the masonry was completed in October 1810. The light was first exhibited to the public on the night of the 1st of February 1811. The difficulties and hazards of this work were chiefly caused by the short time during which the rock was accessible between the ebbing and flowing tides; and amongst the many eventful incidents which render the history of this work interesting, was the narrow escape which the engineer and thirty-one persons made from being drowned, by the rising of the tide upon the rock, before a boat came to their assistance, at a time when the attending vessel had broken adrift. This circumstance occurred before the barrack-house was erected, and is narrated by Mr Stevenson, in his Account of the work, published at the expense of the Lighthouse Board in 1824, to which I would refer for more minute information on the subject of this work and other lighthouses on the coast of Scotland.

The Bell Rock Tower is 100 feet in height, 42 feet in
diameter at the base, and 15 at the top. The door is 30 feet from the base, and the ascent is by a massive bronze ladder. The apartments, including the lightroom, are six in number. The light is a revolving red and white light; and is produced by the revolution of a frame containing sixteen Argand lamps, placed in the foci of paraboloïdal mirrors, arranged on a quadrangular frame, whose alternate faces have shades of red glass placed before the reflectors, so that a red and white light is shewn successively. The machinery, which causes the revolution of the frame containing the lamps, is also applied to tolling two large bells, in order to give warning to the mariner of his approach to the rock in foggy weather. The erection of the Bell Rock Lighthouse cost £61,331, 9s. 2d.

The most remarkable lighthouse on the coast of Ireland is that of Carlingford near Cranfield Point, at the entrance of Carlingford Lough. It was built according to the design of Mr George Halpin, the Inspector of the Irish Lights; and was a work of an arduous nature, being founded 12 feet below the level of high water, on the Hawlbowling Rock, which lies about two miles off Cranfield Point. The figure of the tower is that of the frustum of a cone, 111 feet in height, and 48 in diameter at the base. The light, which is fixed, is from oil burned in Argand lamps, placed in the foci of paraboloïdal mirrors. It was first exhibited on the night of December 20, 1830.
The Skerryvore Rocks, which lie about 12 miles WSW. of the seaward point of the Isle of Tyree, in Argyllshire, were long known as a terror to mariners, owing
to the numerous shipwrecks, fatal alike to the vessels and the crews, which had occurred in their neighbourhood. A list, confessedly incomplete, enumerates thirty vessels lost in the forty years preceding 1844; but how many others, which during that period had been reported as "foundered at sea," or as to whose fate not even an opinion has been hazarded, may have been wrecked on this dangerous reef, which lies so much in the track of the shipping of Liverpool and the Clyde, it would be vain to conjecture.* The Commissioners of the Northern Lighthouses had for many years entertained the project of erecting a lighthouse on the Skerryvore; and with this object had visited it, more especially in the year 1814, in company with Sir Walter Scott, who, in his diary, gives a graphic description of its inhospitable aspect. The great difficulty of landing on the rock, which is worn smooth by the continual beat of Atlantic waves, which rise with undiminished power from the deep water near it, held out no cheering prospect; and it was not until the year 1834, when a minute survey of the reef was ordered by the Board, that the idea of commencing this formidable work was seriously embraced.

The reef is composed of numerous rocks, stretching over a surface of nearly 8 miles from WSW. to ENE. The main nucleus, which alone presents sufficient surface for the base of a lighthouse, is nearly 3 miles from the

\footnotetext{
* The islanders of Tyree made regular trips after storms to the Rock, and often returned with their boats loaded with wreck. Even during the progress of the works, a ship's anchor, and some chain cable much rusted, and other articles, were fished from the hollows of the reef.
}
seaward end of the cluster. It is composed of a very compact gneiss, worn smooth as glass by the incessant play of the waters, and is so small that at high water little remains around the base of the tower but a narrow band of a few feet in widtb, and some rugged humps of rock, separated by gullies through which the sea plays almost incessantly. The cutting of the foundation for the tower in this irregular flinty mass occupied nearly two summers ; and the blasting of the rock, in so narrow a space, without any shelter from the risk of flying splinters, was attended with much hazard.

In such a situation as that of Skerryvore everything was to be provided beforehand and transported from a distance; and the omission in the list of wants of even a little clay for the tamping of the mine-holes, might for a time have entirely stopped the works. Barracks were to be built at the workyard in the neighbouring Island of Tyree, and also in the Isle of Mull, where the granite for the tower was quarried. Piers were also built in Mull and Tyree for the shipment and landing of materials; and at the latter place a harbour or basin, with a reservoir and sluices for scouring the entrance, were formed for the accommodation of the small vessel which attends the lighthouse. It was, besides, found necessary, in order to expedite the transport of the building materials from Tyree and Mull to Skerryvore Rock, to build a steam-tug, which also served, in the early stages of the work, as a floating barrack for the workmen. In that branch of the service she ran many risks, while she lay moored off the rock in a perilous anchorage, with two-thirds of the horizon of foul ground, and a
rocky and deceitful bottom on which the anchor often tripped.

The operations at Skerryvore were commenced in the summer of 1838 , by placing on the rock a wooden barrack, similar to that first used by Mr Robert Stevenson at the Bell Rock. (See Plate II.) The framework was erected in the course of the season on a part of the rock as far removed as possible from the proposed foundation of the lighthouse tower; but in the great gale which occurred on the night of the 3 d of November following, it was entirely destroyed and swept from the rock, nothing remaining to point out its site but a few broken and twisted iron stancheons, and attached to one of them a piece of a beam so shaken and rent by dashing against the rock as literally to resemble a bunch of laths. Thus did one night obliterate the traces of a season's toil, and blast the hopes which the workmen fondly cherished of a stable dwelling on the rock, and of refuge from the miseries of sea-sickness, which the experience of the season had taught many of them to dread more than death itself. After the removal of the roughest part of the foundation of the tower had been nearly completed, during almost two entire seasons, by the party of men who lived on board the vessel while she lay moored off the rock, a second and successful attempt was made to place a second beacon of the same description, but strengthened by a few additional iron ties, \(b b\), and a centre post, oo, in a part of the rock less exposed to the breach of the heaviest waves than the site of the first barrack had been. This second house braved the storm for several years after the works were
finished, when it was taken down and removed from the rock to prevent any injury from its sudden destruction by the waves. Perched 40 feet above the wave-beaten rock, in this singular abode, the writer of this little volume, with a goodly company of thirty men, has spent many a weary day and night at those times when the sea prevented any one going down to the rock, anxiously looking for supplies from the shore, and earnestly longing for a change of weather favourable to the recommencement of the works. For miles around nothing could be seen but white foaming breakers, and nothing heard but howling winds and lashing waves. At such seasons much of our time was spent in bed; for there alone we had effectual shelter from the winds and the spray, which searched every cranny in the walls of the barrack. Our slumbers, too, were at times fearfully interrupted by the sudden pouring of the sea over the roof, the rocking of the house on its pillars, and the spurting of water through the seams of the doors and windows, symptoms which to one suddenly aroused from sound sleep, recalled the appalling fate of the former barrack, which had been engulphed in the foam not twenty yards from our dwelling, and for a moment seemed to summon us to a similar fate. On two occasions, in particular, those sensations were so vivid as to cause almost every one to spring out of bed; and some of the men fled from the barrack by a temporary gangway, to the more stable but less comfortable shelter afforded by the bare wall of the lighthouse tower, then unfinished, where they spent the remainder of the night in the darkness and the cold.

The design for the Skerryvore Lighthouse was given
by the writer of this volume, and was an adaptation of Smeaton's Eddystone Tower to the peculiar situation and the circumstances of the case at the Skerryvore, with such modifications in the general arrangements and dimensions of the building, as the enlarged views of the importance of lighthouses which prevail in the present day seemed to call for. These peculiarities will, however, be noticed incidentally in a subsequent part of this volume which treats of the construction of lighthouse towers; and it will only be necessary in this place to notice a few of the principal dimensions of the building, and some circumstances connected with the work.

The quarries in Tyree having failed to produce an adequate supply of materials for the work, recourse was had to the granite rock of the Ross of Mull, access to which, free of all tax or ground-rent, was, in the most liberal manner, granted by the proprietor, His Grace the Duke of Argyll. This change of operations involved, as already noticed, the cost of a separate establishment in the Isle of Mull, as well as the expense attending the double reshipment of the materials, and their transport from Mull to Tyree, a passage of about 30 miles through a very rough seaway. The produce of the Mull quarries exceeds anything in the modern statistics of granite quarries with which I am acquainted; and those who feel an interest in the subject of quarrying and dressing granite, may find the principal details given between pages 112 and 127 of the Account of the Skerryvore Lighthouse.

The Skerryvore Tower is 138 feet 6 inches high, and 42 feet in diameter at the base, and 16 feet at the top.

It contains a mass of stone work of about 58,580 cubic feet, or more than double that of the Bell Rock, and not much less than five times that of the Eddystone. The lower part of the tower was built by means of jib-cranes, and the upper part with shear-poles, needles, and a bal-ance-crane. The shear-poles were similar to those used by Smeaton at the Eddystone; and the jib-cranes and balance-crane were the same as those which were designed for and first employed by Mr Robert Stevenson, in the erection of the Bell Rock Lighthouse. The bal-ance-crane used at Skerryvore, which was necessarily somewhat larger than that of the Bell Rock, is shewn at Plate III., in which \(a b\) is a portion of a cast-iron pipe or pillar, erected in the centre of the tower, and susceptible of being lengthened as the tower rose, by means of additional pieces of pillar let in by spigot and faucet joints. On this pillar a frame of iron was placed, capable of revolving freely round it, and carrying two trussed arms and a double train of barrels and gearing, worked by men standing on the stages S S, which revolved round \(a b\), along with the framework of the crane from which they hung. On the one arm hung a cylindric weight of cast-iron W, which could be moved along it by means of the gearing, so as to increase or diminish by leverage its effect as a counterpoise; and on the other was a roller \(R\). The roller was so connected with the weight on the opposite arm, as to move along with it, receding from or approaching to the centre pillar of iron in the same manner as the weight did. From the roller hung a sheave, over which a chain moved, with a hook B at the end for raising the stones. When a
stone was to be raised, the weight and the sheave were drawn out to the end of the arms at \(P\) of the crane, which projected over the outside of the walls of the tower, and they were held in their places by simply locking the gearing which moved them. The second train of gearing was then brought into play to work the chain which hung over the sheave, and so to raise the stone to a height sufficient to clear the top of the wall. When in that position, the first train of gearing was slowly unlocked, and the slight declivity inwards from the end of the arms formed an inclined plane, along which the roller carrying the sheave was allowed slowly to move (one man using a break on the gearing to prevent a rapid run), while the first train of gearing was slowly wound by the others, so as to take up the chain which passed over the sheave, and thus to keep the stone from descending too low in proportion as it approached the centre of the tower. When the stone so raised had reached such a position as to hang right over the wall, the crane was made to turn round the centre column in any direction that was necessary, in order to bring it exactly above the place where it was to be set; and by working either train of gearing, it could be moved horizontally or vertically in any way that was required. A needle is merely a beam projecting from the building, with a pulley at its outer end, through which a chain is worked by means of a crab placed inside the tower; it was used for raising the stone to such a level as to be within reach of the chain from the balance-crane on the top of the building.

The mortar used at the Skerryvore was compounded
of equal parts of limestone (from the Halkin Mountain, near Holywell, in North Wales), burnt and ground at the works, and of Pozzolano earth. The mixture was carefully beaten up to the required consistency with seawater. All the joints of each course of the building were carefully filled with grout, which is cement in a fluid state.

The light of Skerryvore is revolving, and reaches its brightest state once every minute. It is produced by the revolution of eight great annular lenses around a - central lamp with four wicks, and belongs to the first order of dioptric lights in the system of Fresnel. It is identical with that which is shewn in Plates IV. and V., and described in the second part of this treatise. The light may be seen from a vessel's deck at the distance of eighteen miles.

The entire cost of the lighthouse, including the purchase of the steam-vessel, and the building of the harbour at Hynish for the reception of the small vessel which now attends the lighthouse, was \(£ 86,977,17 \mathrm{~s} .7 \mathrm{~d}\)., the detailed items of which will be found in the Appendix to the Account of the Lighthouse, already alluded to.

In such a situation as the Skerryvore, innumerable delays and disappointments were to be expected by those engaged in the work; and the entire loss of the fruit of the first season's labour in the course of a few hours, was a good lesson in the school of patience, and of trust in something better than an arm of flesh. During our progress, also, cranes and other materials were swept away by the waves; vessels were driven by sudden gales to seek shelter at a distance from the rocky shores of Mull

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and Tyree ; and the workmen were left on the rock desponding and idle, and destitute of many of the comforts with which a more roomy and sheltered dwelling, and the neighbourhood of friends, are generally connected. Daily risks were run in landing on the rock in a heavy surf, in blasting the splintery gneiss, or by the falling of heavy bodies from the tower on the narrow space below, to which so many persons were necessarily confined. Yet had we not any loss of either life or limb; and although our labours were prolonged from dawn to night, and our provisions were chiefly salt, the health of the . people, with the exception of a few slight cases of dysentery, was generally good throughout the six successive summers of our sojourn on the rock. The close of the work was welcomed with thankfulness by all engaged in it; and our remarkable preservation was viewed, even by many of the most thoughtless, as, in a peculiar manner, the gracious work of Him by whom " the very hairs of our heads are all numbered."

\section*{ON THE CONSTRUCTION OF LIGHTHOUSE-TOWERS.}

In making a design for a Lighthouse-Tower in a situation exposed to the force of the waves, numerous considerations at once present themselves to the engineer ; and it is difficult to assign to any one of them a priority in the train of thought which eventually conducts him to the formation of his plan. These considerations, however, may be conveniently divided into two classes :1st, Those which refer to elements common to lighthouses in all situations, and differ only in amount, such as the height of the tower necessary for commanding a given visible horizon, and the accommodation required for the lightkeepers and the stores; and, \(2 d\), Those which are peculiar to towers in exposed situations, and which refer solely to their fitness to resist the force of the waves which tends to overturn or destroy them. The first class of considerations is so extremely simple, as to require but few remarks in this place. The distance at which it is desirable that a light should be visible being ascertained, with reference to the nature of the surrounding seas and the extent to which any dangerous or foul ground lies seaward of the proposed lighthouse, the height of the tower is at once determined by means of the known relations which subsist between the spheroidicity of the earth, the effects of atmospheric refraction, and the height required for an
object which is to be seen from a given distance. The question regarding the space to be provided in the interior of the tower, can only be properly answered by a person who has a minute practical acquaintance with the peculiar internal economy and management of a lighthouse. The accommodation required for lighthouses in exposed situations must, in a considerable degree, depend upon the greater or less facility of access to them, and the opportunities for replenishing the stores of all kinds which are in daily consumption. In such places, also, the risk of accidents naturally leads to the precaution of retaining additional lightkeepers, and of having duplicates, or even triplets, of those parts of the apparatus that are liable to be injured. Of such circumstances, corresponding extension of the space devoted to the reception of stores and the accommodation of the lightkeepers, is the necessary consequence. In the long nights of winter, when the lamps are kept burning in the northern parts of Great Britain for about seventeen hours, during which time they are never left for a moment without the superintendence of at least one keeper, the care of the light, even in the most favourable situations, necessarily occupies at least two persons; but in places like the Eddystone, the Bell Rock, and the Skerryvore, where it sometimes happens that six or eight weeks elapse without its being possible to effect a landing, it has been thought necessary that there should never be fewer than three keepers on duty. This addition to the ordinary establishment of a lighthouse calls for a greater number of sleeping-cabins, and, at the same time, involves a corresponding increase in the sup-
ply of water, fuel, and other provisions, requiring much additional stowage. So far, therefore, a light-tower in an exposed situation, differs from one on shore only in the extent of its internal accommodation.

The second class of considerations, which must guide the engineer in framing a design for a light-tower which is exposed to the force of the waves, refers solely to the stability of the building.

The first observation which must occur to any one who considers the subject is, that we know little of the nature, amount, and modifications of the forces, on the proper investigation of which the application of the principle which regulates the construction must be based. When it is recollected, that, so far from possessing any accurate information regarding the momentum of the waves, we have little more than conjecture to guide us, it will be obvious, that we are not in a situation to estimate the power or intensity of those shocks to which seatowers are subject; and much less can we pretend to deal with the variations of these forces which shoals and obstructing rocks produce, or to determine the power of the waves as destructive agents. No systematic or intelligible attempt has been made practically to measure the force of the waves, so as to furnish the engineer with a constant to guide him in his attempts to oppose the inroads of the ocean. The only experiments, indeed, on the subject, are those of Mr Thomas Stevenson, civil engineer, who had long entertained the idea of registering the force of the impulse of the waves, and lately contrived an instrument for the purpose, which he has applied at various parts of the coast. The re-
sults he has detailed in the Transactions of the Royal Society of Edinburgh of 20th January 1845. It would naturally be expected, that the force of the waves should vary according to the season of the year and the nature of the exposure; and this expectation is fully justified by the indications of the Marine Dynamometer. Thus it appears, that during five summer months of 1843 and 1844 , the average indications registered at different places near Tyree and Skerryvore, gave 611 lb . of pressure per square foot of surface exposed to the waves; while the average for the winter months for the same places during those two years, gave 2086 lb . per square foot, or upwards of three times that of the summer months. It also appears, that the greatest result as yet obtained at Skerryvore Rock was 4335 lb . per square foot; while that observed on the Bell Rock was 3013 lb., or one-fourth part less than that of Skerryvore. But these experiments have not been continued long enough, nor, as yet, applied to a sufficient variety of places to render them available for the engineer. In the present state of our information, therefore, we cannot be said to possess the elements of exact investigation, and must consequently be guided chiefly by the results of those numerous cases which observation collects, and which reason arranges, in the form which constitutes what is called professional experience. This kind of experience can only be acquired by long habit in carefully observing the appearance and effects of waves in different situations, and under various circumstances. We must attend to their magnitude and velocity, their level in regard to the rocks on which they
break, the height of the spray caused by their collision against the shore, the masses of rock which they have been able to move, and those which have successfully resisted their assault; as also, where such exist, the slopes of the shores produced by the waves, viewed in connection with the nature of the materials composing the beach, with many other transient features which an experienced eye seizes and fixes in the mind as elements of primary importance in determining the power of the sea to produce certain effects. Such phenomena, with all their features and circumstances, we may carry in our recollection; and by comparing them with what has been observed at places where we know that artificial works have resisted the shocks of the waves, we may in some cases successfully arrive at a conclusion as to what works will, at all events, be within the bounds of safety. We must not, however, in any case, venture to approach very near the limit of stability, so long as we continue to labour under our present disadvantages of defective information on some of the most important elements in the inquiry. If it be asked, therefore, how the size and form of buildings exposed to the shock of the waves are to be determined, the answer must be, that, in any given case, the problem is to be solved chiefly by the union of an extensive knowledge of what the sea has done against man, and how, and to what extent, man has succeeded in controlling the sea; together with a cautious comparison of the circumstances which modify and affect any given case which has not been the object of direct experience; nor does it seem possible as yet to found the art of engineering, in so far
as it refers to this class of works, upon any more exact basis. The uncertainty which must ever attend such reasoning can only, it is obvious, be dispelled by actual experience of the result; and time only can test the success of our schemes in all cases of difficulty.

A primary inquiry, as to towers in an exposed situation, is the question, whether their stability should depend upon their strength or their weight; or, in other words, on their cohesion, or their inertia? In preferring weight to strength, we more closely follow the course pointed out by the analogy of nature; and this must not be regarded as a mere notional advantage, for the more close the analogy between nature and our works, the less difficulty we shall experience in passing from nature to art, and the more directly will our observations on natural phenomena bear upon the artificial project. If, for example, we make a series of observations on the force of the sea, as exerted on masses of rock, and endeavour to draw from those observations some conclusions as to the amount and direction of that force, as exhibited by the masses of rock which resist it successfully and the forms which those masses assume, we shall pass naturally to the determination of the mass and form of a building which may be capable of opposing similar forces, because we conclude, with some reason, that the mass and form of the natural rock are exponents of the amount and direction of the forces they have so long continued to resist. It will readily be perceived, that we are in a very different and less advantageous position when we attempt, from such observations of natural phenomena in which weight is solely concerned, to deduce the
strength of an artificial fabric capable of resisting the same forces; for we must at once pass from one category to another, and endeavour to determine the strength of a comparatively light object which shall be able to sustain the same shock, which we know, by direct experience, may be resisted by a given weight. Another very obvious reason why we should prefer mass and weight to strength, as a source of stability, is, that the effect of mere inertia is constant and unchangeable in its nature; while the strength which results, even from the most ju-diciously-disposed and well-executed fixtures of a comparatively light fabric, is constantly subject to be impaired by the loosening of such fixtures, occasioned by the almost incessant tremor to which structures of this kind must be subject, from the beating of the waves. Mass, therefore, seems to be a source of stability, the effect of which is at once apprehended by the mind, as more in harmony with the conservative principles of nature, and unquestionably less liable to be deteriorated than the strength, which depends upon the careful proportion and adjustment of parts.

Having satisfied himself that weight is the most eligible source of stability, the next step of the engineer is, to inquire what quantity of matter is necessary to produce stability, and what is the most advantageous form for its arrangement in a tower. The first question, which respects the mass to be employed, is, as already stated, one of the utmost difficulty, and can be solved by experience alone, directed by that natural sagacity which Smeaton, in his account of his own thoughts on the subject, with much naïveté, terms "feelings," in contradis-
tinction to that more accurate process of deduction which he calls "calculation." It is very difficult, for example, to conceive that the waves could displace a cylindric block of granite, 25 feet in diameter and 10 feet high, which would contain about 380 tons; and we almost feel that they could not do so. If, in order to test the soundness of this expectation, we appeal to such experience as we possess, and apply to the largest vertical section of such a solid, the greatest force yet indicated by Mr Thomas Stevenson's Marine Dynamometer, which, as already stated, is 4335 lb .* per square foot, we shall obtain a pressure of 484 tons, which, being.reduced by one-half \(\dagger\) for the loss of force occasioned by the convexity of the opposing cylindric surface, gives 242 tons, as the greatest force of the waves tending to displace the cylinder. But in the extreme case we have now supposed the solid will be entirely immersed in the water, and its efficient weight will thus be reduced by 140 tons, or the weight of an equal bulk of sea-water; and the remaining weight of 240 tons, by which it will resist the force of the waves, will be almost exactly equal to the pressure which they exert. This imaginary cylinder may, however, be regarded as still within the limits of safety, because the waves could not overturn it, unless their pressure exceeded the weight of the block in a ratio greater than that of its diameter to its height, which in this case is that of 25 to 10 , or \(2 \frac{1}{2}\) times. In order, therefore, to endanger the stability of the solid by overturning it,

\footnotetext{
* Results as high as 6000 lb . have since been ascertained.
\(\dagger\) This reduction seems to be warranted by the results of some experiments of Bossut.
}
the pressure, instead of being 240 tons, must be 600 tons.* We have thus seen, that the cylinder is secure from the chance of being overturned; but we have yet to consider how far it is exempt from the risk of being displaced by the pressure of the waves, causing it to slide along the surface of the rock, owing to deficiency of friction between the two surfaces in contact. The block, for our present purposes, may be regarded as monolithic, on as a mass composed of parts so united by joggles, treenails, and mortar, as to be free from any tendency to disintegration by the force of the waves; and in this case the stability of the cylinder will depend upon the amount of friction opposing the pressure of the waves which tends to produce a sliding movement. It appears, by some experiments of M. Rondelet, \(\dagger\) that the friction of a block of stone sliding on a chiselled floor of rock is equal to \({ }^{\frac{3}{0}}\) ths of its own weight; and we should thus obtain in the present instance 168 tons, as the amount of friction tending to resist the pressure of the waves, which would therefore exert a power superior to that resistance by 74 tons. \(\ddagger\) But this excess of force would be easily neutralized by the adhesion of the mortar and the abutment of the block against the sides of the foundation-pit into which lighthouse-towers in such exposed places are generally sunk in the solid rock.

\footnotetext{
* This is the product of 240 tons, by the ratio of \(2 \cdot 5\).
\(\dagger\) L'art de bâtir.
\(\pm\) The number 168 is \(\frac{{ }^{1}}{}{ }^{7}\) ths of 240 , which is the weight of the cylinder, reduced by the weight of an equal bulk of salt water; and 74 is the excess of 242 tons, the pressure of the waves, above 168 , the amount of friction.
}

When, in addition to these considerations, we learn that the solid frustum, or lower part of the Eddystone Tower, which has weathered so many storms for the last ninety years, does not much exceed in mass the imaginary cylindric block which I have spoken of, our confidence in the stability of the cylinder is greatly increased. Our belief receives a still farther confirmation from the fact, that the strongest instance recorded of the power of the waves, falls considerably short of the case which I have just imagined. The instance alluded to is given in Mr Lyell's Geology, on the authority of the Reverend George Low, of Fetlar, in Zetland, who mentions, that a block, whose dimensions seem to give us reason to estimate its weight at nearly 300 tons (or about one-fifth less than that of the cylinder), was moved over a point, and thrown into the sea ; and it must be remembered, that the form of this block, which was only 5 feet thick and about 40 feet long, rendered it very susceptible of a sliding motion, and must have greatly aided its transport. We may therefore not unreasonably conclude, that, in designing such a tower, it is safe to assume a mass which our own judgment and recorded facts seem to concur in pronouncing beyond the power of the greatest waves, as fixing the lowest limit to which the contents of the proposed edifice may be reduced.

There are several circumstances, however, which tend to increase or diminish the stability of the same mass exposed to the same forces. Of these a very prominent one is the form of the mass, which may be so modified as to offer more or less resistance to the forces which assault the building. Thus a parallelopiped would be a
much less suitable form for a sea-tower than a cylinder, and so proportionally of all the polygonal prisms which may occur between these two extremes. I remember having heard it proposed, in the course of conversation, by a non-professional friend, that lighthouse-towers might be formed in such a manner, that each horizontal section should be a wedge with its narrow end directed to the greatest assaulting force. This notion is in itself not destitute of ingenuity; for, if the circumstances to which it is to be adapted were constant, we should thereby present the form of least resistance, and, at the same time, the greatest depth and strength of the building to the line of greatest impulse. But the notion is wholly impracticable, because the direction of the winds and waves is so variable, as to render it almost certain that a tower so constructed would, on some occasion, be assaulted in the line of its thinnest section ; and thus, what might in one case be an advantage, would, in the event of such a change in the point of attack, become a great source of weakness, as the flat side of the wedge would then be opposed to the force, thereby presenting to the direct assault of the waves the largest surface, with, at the same time, the most disadvantageous disposition of the resisting matter. There' seems little reason, therefore, for any doubt as to the circular section being practically the most suitable for a tower exposed in every direction to the force of the waves.

Next to this, and hardly to be separated from it, inasmuch as it involves the question regarding the form of the tower, is the position of the centre of gravity. Of any solid viewed as monolithic, it may be said that its ultimate
stability, by which I would understand its resistance to \({ }^{-}\) the final effort which overturns it, will greatly depend upon its centre of gravity being placed as low as possible; and the general sectional form which this notion of stability indicates is that of a triangle. This figure revolving on its vertical axis, must, of course, generate a cone as the solid, which has its centre of gravity most advantageously placed, while its rounded contour would oppose the least resistance which is attainable in every direction. Whether, therefore, we make strength or weight the source of stability, the conic frustum seems, abstractly speaking, the most advantageous form for a high tower. But there are various considerations which concur to modify this general conclusion, and, in practice, to render the conical form less eligible than might at first be imagined. Of these considerations, the most prominent theoretically, although, I must confess, not the most influential in guiding our practice, is, that the base of the cone must in many cases meet the foundation on which the tower is to stand, in such a manner, as to form an angular space in which the waves may break with violence. The second objection is more considerable in practice, and is founded on the disadvantageous arrangement of the materials, which would take place in a conic frustum carried to the great height which, in order to render them useful as sea-marks, lighthouse-towers must generally attain. Towards its top the tower cannot be assaulted with so great a force as at the base, or, rather, its top is entirely above the shock of heavy waves; and, as the diameter of the conoidal solid should be proportioned to the intensity of the shock
which it must resist, it follows that, if the base be constructed as a frustum of a given cone, the top part ought to be formed of successive frusta of other cones, gradually less slanting than that of the base. But it is obvious, that the union of frusta of different cones, independently of the objection which might be urged against the sudden change of direction at their junction, as affording the waves a point for advantageous assault, would form a figure of inharmonious and unpleasing contour, circumstances which necessarily lead to the adoption of a curve osculating the outline of the successive frusta composing the tower; and hence, we can hardly doubt, has really arisen, in the mind of Smeaton, the beautiful form which his genius invented for the lighthouse-tower of the Eddystone, and which subsequent engineers have contented themselves to copy, as the general outline which meets all the conditions of the problem which they have to solve. And here I cannot help observing, as an interesting, and by no means unusual, psychological fact, that men sometimes appear to be conducted to a right conclusion by an erroneous train of reasoning; and such, from his "Narrative," we are led to believe, must have been the case with Smeaton in his own conception of the form most suitable for his great work. In the "Narrative" (§81), he seems to imply, that the trunk of an oak was the counterpart or antitype of that form which his (§ 246) "feelings rather than calculations," led him to prefer. Now, there is no analogy between the case of the tree and that of the lighthouse, the tree being assaulted at the top, and the lighthouse at the base ; and although Smeaton goes on, in
the course of the paragraph above alluded to, to suppose the branches to be cut off, and water to wash round the base of the oak, it is to be feared the analogy is not thereby strengthened; as the materials composing the tree and the tower are so different, that it is impossible to imagine that the same opposing forces can be resisted by similar properties in both. It is obvious, indeed, that Smeaton has unconsciously contrived to obscure his own clear conceptions in his attempt to connect them with a fancied natural analogy between a tree which is shaken by the wind acting on its bushy top, and which resists its enemy by the strength of its fibrous texture and wide-spreading ligamentous roots, and a tower of masonry, whose weight and friction alone enable it to meet the assault of the waves which wash round its base; and it is very singular, that, throughout his reasonings on this subject, he does not appear to have regarded those properties of the tree which he has most fitly characterized as "its elastieity," and the " coherence of its parts." One is tempted to conclude that Smeaton had, in the first place, reasoned quite soundly, and arrived, by a perfectly legitimate process, at his true conclusion ; and that it was only in the vain attempt to justify these conclusions to others, and convey to them conceptions which a large class of minds can never receive, that he has misrepresented his own mode of reasoning. In the paragraph preceding that which refers to the tree (§80), he has, in point of fact, clearly developed the true views of the subject; and, with the single exception of the allusion to the oak, he has discussed the question throughout in a masterly style.

In a word, then, the sum of our knowledge appears to be contained in this proposition-That, as the ultimate stability of a sea-tower, viewed as a monolithic mass, depends, ceteris paribus, on the lowness of its centre of gravity, the general notion of its form is that of a cone; but that, as the forces to uhich its several horizontal sections are opposed decrease towards its top in a rapid ratio, the solid should be generated by the revolution of some curve line convex to the axis of the tower, and gradually approaching to parallelism with it. And this is, in fact, a general description of the Eddystone Tower devised by Smeaton.

It is deserving of notice, as one of the many proofs which the records of antiquity afford of the similarity of the results of human thought in all ages, and of the truth of the Wise Man's saying, that " there is nothing new under the sun," that the ancient Egyptians appear to have had the same conceptions of the solid of stability that were present to the mind of the modern engineer of the Eddystone Lighthouse. In Sir J. Gardner Wilkinson's admirable work on the Manners and Customs of the Ancient Egyptians, he gives, in the first volume of his second series, at page 253, a woodcut shew- Fig. 1 . ing the figure of the deity Pthah, under the symbol of stability, according to Egyptian conceptions. This symbol so closely and strikingly resembles the general appearance of the Eddystone, that I willingly give it a place in the text (Fig. 1),
 denuded, however, of the arms and head-dress of the deity whom it shrouds.

In illustrating the application of these general no-
tions to practice, I shall perhaps be excused for adopting the most natural course of referring chiefly to my design for the lighthouse-tower on the Skerryvore Rock, in forming which I was guided by numerous circumstances, which modified my views, and produced the individual form of tower which I have adopted. Since the days of Smeaton, when his magnificent tower was lighted by common candles, the application of optical apparatus to lighthouses has greatly altered the state of the case; and the improvement of the system in modern times has, in most instances, rendered a greater altitude of tower desirable, in order to extend as much as possible the benefit of a system capable of illuminating the visible horizon of any tower which human art can reasonably hope to construct. In the particular case of the Skerryvore also, the great distance of the outlying rocks (some of which are 3 miles right seaward of the lighthouse), concurs with the improvement of the lights, in making it desirable that the tower should be of considerable height, and that the light should command an extensive range. It was, therefore, from the first consideration of the subject, determined that the light should be elevated about 150 feet above high water of spring tides, so as to illuminate a visible horizon of not less than 18 miles of radius; and, after much deliberation, and a full consideration of the infrequency of communication with the proposed lighthouse, from the great difficulty of landing on the rock, and the consequent uncertainty of keeping up the supplies, I found that, for the convenient accommodation of the lightkeepers and the suitable stowage of the stores, a void space of about

13,000 cubic feet would be required. These elements being fixed, the general proportions of the tower came next to be considered.

In the Eddystone, the radius of the base, at the level of high water of spring tides, is somewhat less than onefifth of the height of the tower above that level ; while in the Bell Rock, at the same level, it is little more than one-seventh of the height. If, again, we suppose the curve of the Eddystone to be continued downwards to the level of low water, the radius (in so far as we may judge from sketching the continuation of a curve undefined by any geometrical property) would be rather more than one-fourth of the whole height above that level ; while in the Bell Rock the proportion, in reference to the same level, is a little more than one-fifth. Viewing the whole height of the Skerryvore Tower above high water of spring tides as equal to 142 feet, and finding that, in the cases of the Eddystone and the Bell Rock, the radius of the horizontal section at that level is respectively one-fifth and one-seventh of the whole height; and again, viewing the extreme height of the Skerryvore Tower above low water of spring tides as equal to about 155 feet, and considering the proportionate radii of the Bell Rock and Eddystone (in so far as the latter is ascertainable) as respectively one-fifth and one-fourth of the heights of the top of the masonry above the level of low water, I finally decided upon giving the tower at the Skerryvore such dimensions as would not be widely discordant with these general proportions. In this view, I determined that the radius of the base should not exceed 22 feet, on the level of about 4 feet above the high
water mark, where I expected to obtain a solid founda-tion-a base which bears to the whole height of the tower a proportion somewhat less than that of the Bell Rock, which is one-fifth. It so happens, that the diameter adopted is nearly the greatest which the rock affords; for I found, after many careful examinations of the gullies and fissures which intersect it, that some of the concealed cavities ran much farther into the rock than \(I\) at first had imagined. The adoption of a much larger base, even had it been otherwise advisable, would therefore have involved some risk of the external ring of stones, of the lowest course, giving way by the yielding of an unsound part of the outer portion of the rock, to the pressure of the superincumbent mass, and might eventually have led to the destruction of the tower.

The height of the pillar having been finally fixed at 138.5 feet, and the radius of the base, at the level of about 4 feet above high water, at 21 feet, I next proceeded to consider the details of its proportions. Of the whole height of 138.5 feet, 18 were to be absorbed in a suitable capital for the pillar, consisting of a parapet for the lantern, an abacus, a cavetto, and a belt separating these from the shaft. The internal void I determined should be 12 feet in diameter, as the size most suitable for the reception of the lantern and apparatus; and this, combined with the choice of about \(13,000 \mathrm{cu}-\) bic feet of void already mentioned, fixed the height of the solid frustum at the base of the tower at about 26 feet above the foundation. Having farther decided that the thinnest part of the walls, immediately under the
belt-course which separates the capital from the shaft, should not be less than 2 feet thick, as necessary to give due solidity and strength to the walls, and prevent, by the breadth of the joints, the percolation through the walls of the water which might be furiously dashed against them in storms, I had nothing farther to do but to determine the nature of the line which should connect the extremities of the top and bottom radii of the pillar. As I had already concluded that this line must, as in the Eddystone and Bell Rock, be a curve line, concave to the sea, I next proceeded to try the effects of various curves traced between these points, in giving a convenient and advantageous disposition of the materials, both for the due thickness of the walls and the mass of the solid frustum at the base of the tower. These two points, as will be better understood by means of the accompanying diagram (Fig. 2), are separated from each other vertically 120.25 feet, and are horizontally distant from each other 13 feet, which is the excess of the bottom radius over that of the top of the shaft, or the consequent amount of what may be called the aggregate slope of the wall. The solid generated by the revolution of some curve
 line about the vertical axis of the building thus becomes the shaft of the pillar. For this purpose I tried four different curves, the parabola, logarithmic, hyperbola, and conchoid, in all of which the level of the centre of gravity of the mass, which was carefully computed, varied but little from 30 feet above the base.

The logarithmic curve I at once rejected, from its too near approach to a conic frustum, and the excessive thickness of the walls which such a figure would produce, where the hollow cylindric space for the internal accommodation commences at the level of 26 feet above the base. The parabolic form displeased my eye by the too rapid change of its slope near the base ; and I had moreover some difficulty in reconciling myself to the condition of the exterior ring of stones at the base, too much of the outer portion of each stone being left without the advantage of direct pressure from the superincumbent mass of the wall above. The two remaining pillars, derived from the hyperbolic and conchoidal frusta, are nearly identical in form ; and of these two curves I preferred the former, which gives the most advantageous arrangement of materials, in regard to stability, of all the four forms. This quality of advantageous proportion exists in those four curves in the ratio of the numbers in the last column of the following table;* which shews a

\footnotetext{
* The last column of this table is derived as follows:-Assuming that the economic advantage of any proposed tower of given height and diameter at base and top, is inversely as the mass and the height of the centre of gravity above the base, and denoting these quantities by \(M\) and \(G\) respectively, the fraction \(\frac{1}{G \cdot M}\) may be taken as an indication of the economic advantage of the proposed tower. Let \(\frac{1}{G^{\prime} \cdot \mathbf{M}^{\prime}}\) express the economic advantage of another tower; then the advantage of the second tower, compared to that of the first, taken as unity, will be \(\frac{G \cdot M}{G^{\prime} \cdot M^{\prime}}\), by which expression the last column in the table was calculated.
}
slight superiority of the hyperbolic over any of the other forms.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Hypothetical
fowers. & Height of the Tower (H.) & \[
\begin{gathered}
\text { Diam } \\
\text { st } \\
\text { Base } \\
\text { in } \\
\text { feet. }
\end{gathered}
\] & \begin{tabular}{l}
eter \\
Top \\
\({ }^{\text {in }}\) \\
feet.
\end{tabular} & Volume of solid Tower in cubic feet. M. & Distance of centre of Grarity from Base. \(\Theta\). & \(\stackrel{H}{\mathbf{H}}\) & Economic Advantage.
\[
\frac{\mathbf{G} \cdot \mathbf{M}^{\prime}}{\mathbf{G}^{\prime} \cdot \mathbf{M}^{\prime}}
\] \\
\hline Hyperbolic, & 120 & 42 & 16 & 62,915 & \(41 \cdot 227\) & 2.911 & 1.00000 \\
\hline Conchoidal, & 120 & 42 & 16 & 62,984 & 41.336 & 2.903 & 0.99627 \\
\hline Parabolic, & 120 & 42 & 16 & 63,605 & 43.400 & 2.765 & 0.93963 \\
\hline Logarithmic, & 120 & 42 & 16 & 74,742 & 42.460 & 2.826 & 0.81608 \\
\hline Conical, & 120 & 42 & 16 & 84,737 & 43.280 & \(2 \cdot 773\) & 0.70725 \\
\hline
\end{tabular}

The shaft of the Skerryvore pillar, accordingly, is a solid, generated by the revolution of a rectangular hyperbola about its assymptote as a vertical axis. Its exact height is \(120 \cdot 25\) feet, and its diameter at the base 42 feet, and at the top 16 feet. The ordinates of the curve at every foot of the height of the column, were carefully determined in feet to three places of decimals; and from them the working drawings for the moulds of the stones were made at full size. The first 26 feet of height is a solid frustum, containing about 27,110 cubic feet, and weighing about 1990 tons.* Immediately above this level the walls are 9.58 feet thick, whence they gradually decrease throughout the whole height of the shaft, until, at the belt, they are reduced to 2 feet in thickness. Above the shaft rests a cylindric belt 18

\footnotetext{
* At the rate of 13.62 cubic feet of granite to a ton.
}
inches deep; and this is surmounted by a cavetto 6 feet high, and having 3 feet of projection. The contour of this cavetto is that resulting from a quadrant of an ellipse revolving about the centre of the tower, with a radius of 8 feet on the level of its transverse axis; and the moulds for this curve were drawn at full size from co-ordinates calculated for the purpose. The cavetto supports an abacus 3 feet deep, the upper surface of which forms the balcony of the tower, and above it rest the parapet-wall and lantern.

It may, perhaps, be not uninteresting to the reader to examine Plate I., which shews, on one scale, the elevations of the lighthouses of the Eddystone, the Bell Rock, and the Skerryvore, and exhibits the level of their foundations in relation to high water. It will also serve to give some idea of the proportionate masses of the three buildings. The position of the centre of gravity, as calculated from measurements of the solids, is also marked by a round black dot and letter \(G\) on each tower ; and in the table following, I have given the cubic contents of each of those towers, the height of the centre of gravity above the base, and the ratio of that quantity to the height of the tower.
\begin{tabular}{|l|c|c|c|c|c|c|}
\hline Lighthouse. & \begin{tabular}{c} 
Height of \\
Tower above \\
frist entire \\
course. \\
(H.)
\end{tabular} & \begin{tabular}{c} 
Contents \\
of Tower.
\end{tabular} & \multicolumn{2}{|c|}{\begin{tabular}{c} 
Diameter \\
at Base.
\end{tabular}} & \begin{tabular}{c} 
Distance of Top. \\
centre of \\
gravity in \\
faet from \\
Base. \\
(G.)
\end{tabular} & \(\frac{\mathbf{H}}{\mathbf{G}}\) \\
\hline Eddystone, & 68 & 13,343 & 26 & 10 & 15.92 & 4.27 \\
Bell Rock, & 100 & 28,530 & 42 & 15 & 23.69 & 4.24 \\
Skerryvore, & 138.5 & 58,580 & 42 & 16 & 34.95 & 3.96 \\
\hline
\end{tabular}

Lastly, I will briefly notice the few subordinate points in which the design of the Skerryvore Tower may be regarded as differing from those of the Eddystone and the Bell Rock. In glancing at the contrasted figures of the three buildings, it will be at once observed that the outline of the Skerryvore approaches more nearly to that of a conic frustum than the other two. To the adoption of this form, various considerations induced me; and these I shall very briefly detail. In the first place, it seemed to me that, both in the Bell Rock and the Eddystone, the thickness of the walls had been reduced to the lowest limits of safety towards the top; and experience has shewn that heavy seas and winds acting upon a weighty cornice, cause a degree of tremor which I felt satisfied would not occur in a building with thicker walls. The effect of thickening the walls at the top, is, of course, cateris paribus, to diminish the projection of the base, and thus to produce less concavity of figure, and consequently a nearer approximation to the contour of a conic frustum. I have already stated, that this excess of the bottom radius over that of the top, is in the Skerryvore Tower 13 feet, and that the height of the shaft is 120.25 feet. The quotient resulting from the division of the height by the excess of the bottom radius over that at the top is 9.27 ; and, if the figure had been conical, this number would have given a measure of the slope of the walls throughout. There can be little doubt that the more nearly we approach to the perpendicular, the more fully do the stones at the base receive the effect of the pressure of the superincumbent mass as a means of retaining them in their places, and the more per-
fectly does this pressure act as a bond of union among the parts of the tower. This consideration materially weighed with me in making a more near approach to the conic frustum, whioh, next to the perpendicular wall, must, other circumstances being equal, possess the property of pressing the mass below with a greater weight, and in a more advantageous manner, than a curved outline in which the stones at the base are necessarily farther removed from the line of the vertical pressure of the mass at the top.* This vertical pressure operates in preventing any stone being withdrawn from the wall in a manner which, to my mind, is much more satisfactory than the effect of an excessive refinement in dovetailing and joggling, appliances which I consider as chiefly useful in the early stages of the progress of a work, before the superstructure is raised to such a height as to prevent heavy seas from breaking right over it.

If these views be substantially correct, it may not, perhaps, be altogether inadmissible (without, however, venturing to enunciate any general law) to conclude,

\footnotetext{
* It is most satisfactory to find that the views expressed above, regarding the eligibility of the conical form, seem to have the sanction of the late Dr Thomas Young, who appears to have conneeted his preference of this form with its greater efficiency as a source of friction among the parts of a building. In his syllabus of Lectures, under the section "Architecture," he thus speaks : "For a lighthouse where a great force of wind and water was to be resisted, Mr Smeaton chose a curve convex to the axis. In such a case, the strength depends more on weight than on cohesion, and also in a considerable degree on the frietion which is the effect of that weight. Perhaps a cone would be an eligible form."
}
that, in the three lighthouses of the Eddystone, the Bell Rock, and the Skerryvore, this source of union among the outer stones of the lower courses must bear some proportion to the numbers 753,659 , and 927 , which are the quotients of the height of the column, divided by the difference of the top and bottom radii of the shaft, in each case respectively. This consideration seems too important to be entirely overlooked; and I conceive that, by following out this view, I have been enabled to depart with perfect safety from the intricate and elaborate work required for the connection of the materials by means of dovetailing and joggling, which the adoption of a more concave outline (in which the vertical pressure could not have been so advantageously transmitted to the outer stones of the base), would perhaps have rendered advisable. In the case of the Bell Rock, however, whose construction, in regard to this property, is the least advantageous of the three buildings, it must be borne in mind that the tower is covered to the depth of 15 feet at spring tides, and that this principle of verticall pressure could not have been safely appealed to during the whole of the time which intervened between the commencement of the building and the attainment of a height sufficient to render it available, a period which, in a tower having so great a part submerged, was of necessity much more prolonged than in the other cases. The stones were thus exposed to the full effect of heavy seas, at all levels, during two entire winters, and could not therefore have been safely left, without being kept together by numerous ties and dovetails. It also seemed important, in designing that tower, with
reference to the rise of tide, to give its lower part a sloping form, as the least likely to obstruct the free passage of the waves. The outer stones of the lower courses were also selected of unusual length inwards, so as to bring them more under the influence of the vertical pressure of the upper wall.

Before leaving this subject, I may remark, that it is quite possible to construct a tower of a curved form, in such a manner, that the pressure of the upper part of the pillar shall be distributed to the greatest advantage on every stone, by building the outer walls as inverted arches, so that the section of each stone shall be that of a voussoir, with joints perpendicular to the successive tangents of the curve. This arrangement of the stones is, in fact, practised in sea-walls of various kinds, and has even been recommended for circular towers in an ingenious paper in the Transactions of the Royal Scottish Society of Arts. But in many situations, and at Skerryvore in particular, this mode of transmitting the pressure, so as to throw it perpendicular to the beds of the stones, is inadmissible, as conducing to or involving a greater evil. The evil has already been noticed, and consists in the thrust of the lowest stone (which is of course inclined to the horizon) having a tendency to push out the sides of the rock on which the tower is built. This fear, where the towers are to be placed on small steep rocks or pinnacles, and more especially when these rocks are traversed by veins nearly vertical, is by no means visionary ; and there is good reason to apprehend, that the pressure thus resulting in a line considerably inclined to the plane of cleavage, might throw
outwards a thin portion of rock, which, under the more conservative influence of a vertical pressure, might continue to retain its connection with the rest of the rock unimpaired for ages.

Another method of, in some degree, increasing the resistance of a sea-tower to a horizontal thrust, if such aid be required, is to give the line of courses a continuous spiral form, instead of building them in successive horizontal layers. Were there reason to fear that the entire dislocation of the building might take place in a plane nearly horizontal, this method seems more calculated to counteract the danger than the use of dowels or joggles passing from the course below to the course above; but, as this is one of the accidents least to be apprehended, there does not seem any good ground for resorting to a mode of structure which would lead to great intricacy of workmanship, and would, in practice, be attended with difficulty in obtaining a proper vertical bond or union among the several stones.

The only remaining point, in which the example furnished by the Eddystone and Bell Rock Lighthouses has been at all materially departed from in the Skerryvore, is (as has already been hinted at by an unavoidable anticipation) the mode of uniting the different parts of the masonry together. In both those towers the stones were dovetailed throughout the buildings, chiefly (at least in the case of the Bell Rock, where the foundation was so much below the tide) with the view of preventing the sea from washing away the courses which might be left exposed to the winter storms before the weight of the superstructure had been brought to bear upon them. In the
upper part of the Bell Rock Mr Stevenson also introduced a kind of band joggle, which consists of a flat ribband of stone raised upon the upper bed of one course, and fitting into a corresponding groove cut in the under bed of the course above; and this system of tying the adjoining courses together also forms a chief feature in his design for a lighthouse on the Wolf Rock.* When the great pressure of the superstructure of such towers, however, and the effect of the mortar, are considered, there seems little probability of one course being dislocated, in defiance of the friction resulting from the weight of the column. An impulse sufficient to produce such an effect would tend to overset the whole superstructure from mere deficiency in weight, and in this case the joggle would have little effect. But if joggles be thought necessary for this purpose, the ribband-form certainly produces a better arrangement than that of the cubic joggles employed by Smeaton for connecting the adjoining courses of his building together, as the sectional strength of those scattered square joggles is very small compared to the effect of a shock which could be supposed capable of moving the whole mass of a tower. In the lower parts of the Skerryvore Tower, I entirely dispensed with dovetailing and joggles between the courses, and thus avoided much expensive dressing of materials. The stones were retained in their places during the early progress of the work, chiefly by common diamond joggles, and the courses were temporarily united to each other by wooden treenails, like those used in the Eddy-

\footnotetext{
* Account of the Bell Rock Lighthouse, Plate XXI.
}
stone and Bell Rock. The treenails had split ends, with small wedges of hardwood loosely inserted, which, being forced against the bottom of the holes in the course below, into which the treenails were driven, expanded their lower ends until they pressed against the sides of the holes; while their tops were made tight by similar wedges driven into them with a mallet from above. I have, however, adopted the ribband-joggle in the higher part of the tower, where the walls begin to get thin, in the very same manner as at the Bell Rock, where it was used, partly, that it might counteract any tendency to a spreading outwards of the stones, and partly that it might operate as a kind of false joint to exclude the water which, when pressed with great violence against the tower, is apt to be forced through a straight or plain joint. The stones in the higher courses throughout each ring are also, as in the Bell Rock, connected at the ends by double dovetailed joggles, which unite the two adjoining stones ; and the walls are, besides, tied together, as in that structure, at various points by means of the floor stones, which are all connected by dovetails let into large circular stones forming the centres of the floors. I also ventured to leave out the metallic ties at the cornice, which consisted, at the Eddystone, of chains, and, at the Bell Rock, of a very strong hoop of copper. The reasons which induced me to adopt this change I need not here enlarge upon. It is sufficient to state, that I believe I have nearly balanced the forces which would have tended to throw the cornice outwards, had a greater disproportion existed in the weight of the outer and inner parts of the cavetto, and to mention
that the lightroom or highest floor occurs, at such a level, as of itself to answer all the ends which metallic ties could have served.

In the erection of lighthouses on land, the chief points which demand the attention of the engineer are the provision of a commodious space for the lantern, and also the height of the tower required for a given range; which, from the level of the land where light-towers are placed, is not generally great. The convenient arrangement of the lantern, and the dwelling-houses for the lightkeepers, so as best to serve the purposes of the due maintenance of the light, both in cleaning the apparatus and watching the lamps, are subjects of much practical importance; but they will be more conveniently noticed in another part of this volume, especially in connection with the views in Plates XI. and XII. of the Ardnamurchan Lighthouse.

There are various other lighthouses, which, in themselves, are sufficiently deserving of a separate notice, were it not that they have more or less something in common with those already described, which are unquestionably the most remarkable edifices of the kind.

The first design for an Iron Lighthouse is that of Captain Brodie, R.N., for the Bell Rock, and the next, by Mr Robert Stevenson, for the same situation, in the year 1800.* Mr Stevenson's design consisted of eight cast-iron pillars arranged in the form of a pyramid, and tied together by means of horizontal and diagonal braces, and also attached to a central socket, by

\footnotetext{
* Account, Plate VII., figs. 2, 3, 4, and 5, and pp. 497 and 500.
}
means of diagonal radiating ties. The lower portion of the habitable part was designed in the form of an inverted cone, in order to lessen the shock of the sea from beneath. On this general plan Mr Stevenson, in 1821, erested the Carr Rock Beacon ; and a similar construction has been extensively used for beacons all round the coast of Scotland. Of one of them a view is given in Plate XIII. Messers Walker and Burgess have also designed and partly erected a lighthouse for the Bishop's Rock, in the Scilly Isles, 120 feet high, on this general p.an ; and Mr Lewis of Philadelphia is engaged with the execution of a similar and equally bold design for a lighthouse on the Carysfort Reef, East Florida. Such structures for situations of great exposure, would unloubtedly be less costly than those of masonry; but their stability confessedly depends upon their strength rather than their weight, and, in many situations, it might ultimately be resolved into the security of the fixtures which attaches them to the rock. There is, undoubtedly, reason to fear, that, in structures of great height, urged by the continued force of the waves and winds, a gradual deterioration of the fixtures may result from long-continued tremor, or that the pillars might suddenly sustain very serious injury from the impact of heavy bodies thrown against them by the waves; and while, in proportion to the difficulty of the situation, the expense of the stone structure would increase, so the undesirableness of being forced to renew any structure would increase also; and thus, while in many such places the iron lighthouse may be the most convenient and least expensive in prospect, the stone tower is the more satis-
factory and more durable when completed. I should therefore, in general, say that nothing short of absolute necessity can justify a preference of iron over stone for the purposes of a lighthouse exposed, like the Eddystone, the Bell Rock, and the Skerryvore, to the full fury of the waves. This general opinion is not, of course, intended to reflect upon the plans either of Mr Lewis or of Messrs Walker and Burgess, with the circumstances of which I am quite unacquainted. The gradual change of castiron by the action of the marine acid forms another element in the question as to the fitness of such structures for rocks washed by the sea.

Mr Alexander Gordon of London has fitted up several lighthouses composed of cast-iron plates, on the plan proposed by the late Captain Sir Samuel Brown, R.N., a style of building in itself by no means eligible, and which seems suitable only where stone cannot be easily obtained, or conveniently applied. Such a structure, as well from the difficulty of obtaining permanent fixtures above alluded to, as from its offering more resistance to the waves than can be fully counterbalanced by the inertia produced by filling the lower part of the void with concrete, is, I conceive, but ill adapted for rocks exposed to the direct impact of heavy waves. But a still more formidable objection may be found in its liability to sustain serious injury from floating spars, or rolling boulders of great dimensions ; and this construction, when proposed by Sir Samuel Brown, for the Skerryvore, was, after due deliberation with Messrs William Cubitt and George Rennie, finally rejected by the Lighthouse Board in terms of my report.

Mr Gordon, in a letter lately addressed to Mr Hume, M.P., advocates the suitableness of an iron lighthouse for the Skerryvore Rock, which he has never seen, and which he unreasonably compares with a Rock in Simon's Bay, near the Cape of Good Hope, which also he has never visited, but on which he undertakes to build a lighthouse for one-tenth of the cost of that on the Skerryvore. Now although the Skerryvore Lighthouse cost nearly \(£ 87,000\), it must be borne in mind that fully \(£ 23,000\) of that sum was made up of items which would be common to any lighthouse in that position, such as the establishment at Hynish, in the island of Tyree, consisting of a harbour for the attending vessel, and the houses for the lightkeepers and seamen, and a signal tower ashore. Let, therefore, this sum of common outlay be added to Mr Gordon's random estimate of \(£ 20,000\), and the result is \(£ 43,000\), giving an anticipated numerical saving of \(£ 44,000\), or about one-half instead of nine-tenths. After a residence on the Skerryvore Rock of five seasons, and eleven years' experience of its exposure, I may be allowed to speak with some confidence on this subject; and I shall therefore briefly enumerate a few of the leading facts which bear upon the fitness of an iron structure for that situation. 1. A temporary barrack, similar in construction to that which stood so long on the Bell Rock and afterwards for several seasons at the Skerryvore itself, was carried away in one night. 2. Stones, some of which weighed as much as five tons, were swept by the waves over the top of the rock; and much floating wreck-timber has been seen to pass close to it. 3. The force of the waves, as indicated
by the marine dynamometer has amounted to 6000 lb . per square foot, or twice that at the Bell Rock. 4. Two iron beacons were successively destroyed on the Bo-Pheg Rock in Hynish Bay, 12 miles landward of the Skerryvore Rock, one of which was of the pillar form above noticed, and the other was a cone of iron plates, like that proposed by Mr Gordon, having the lower part of the void filled. Before the plate-beacon was carried away, a hole of 2 feet in diameter was broken through one of the plates,* most probably by a heavy spar urged end on by the waves. Bearing, therefore, all these facts in mind, I have no hesitation in saying that no pecuniary consideration could, in my opinion, have justified the adoption of an iron lighthouse for the Skerryvore. In such a situation, where the establishment has been unvisited for weeks at a time and where I have myself been detained prisoner by the weather in the temporary barrack for a fortnight, even in the month of June, neither the required stability nor the extent of stowage for provisions and stores which are consistent with a due regard to the safety of human life and the certain and regular exhibition of the light, could have been obtained in an iron lighthouse. There are, however, situations which, although difficult of access, are yet removed from the most powerful action of the sea, for which the lightness of the walls and the rapidity of construction may render this plan preferable to a building of stone. For beacons on rocks in such positions, I have often employed this

\footnotetext{
* The plate broken was rather upwards of one inch thick, and was strengthened by means of webbed flanges 6 inches deep.
}
mode of construction, filling the lower part of the inclosed void with concrete.

The useful invention of Mr Mitchell of Belfast, for applying the principle of the screw to the erection of lighthouses on soft foundations, deserves a longer notice than can be given here. It must therefore be sufficient to say, that the principal lighthouses on this plan (those of the Maplin, Fleetwood, and Belfast Lough) consist of piles or of hollow pillars of cast-iron, grouped together in the form of a truncated pyramid, and resembling, in the general arrangement of their parts, the Beacon shewn in Plate XIII. The lower end of each pillar is furnished with a flat screw or worm and a sharp point, which is screwed into the sand, clay, or gravel, or other soft subsoil. Dr Potts has also invented a method of driving piles by means of atmospheric pressure, which has been used at the South Galloper Beacon.

\section*{ON THE ILLUMINATION OF LIGHTHOUSES, AND THE SOURCES OF THE LIGHT.}

Having thus attempted to describe the most interesting and celebrated lighthouses, I proceed to consider the various methods now in use for the illumination of lighthouses. There can be little doubt, that down to a very late period, the only mode of illumination adopted in the lighthouses, even of the most civilised nations of Europe, was the combustion of wood or coal in chauffers, on the tops of high towers or hills. It now seems strange, when late improvements in lighthouse illumination are considered, to hear that so lately as the year 1816, when the Isle of May light, in the Frith of Forth, was assumed by the Commissioners of the Northern Lights, it was of that rude description, and had shewn a coal fire for 181 years since 1635: and even in England, the art of illumination had made so little progress, that the magnificent Tower of the Eddystone, for about forty years after it came from the hands of Smeaton, could boast of no better light than that derived from a few miserable tallow candles; nay, so lately as the year 1801, the light at Harwick, in addition to the coal-fire, had a flat plate of rough brass on the landward side, to serve as a reflector! Such methods were most imperfect, not only in point of efficiency and power, but also
as respects the distinction of one light from another, an object which, on a difficult and rugged coast, may be considered as of almost equal importance with the distance at which the lights can be seen.
Solid substances which remain so throughout their combustion, are only luminous at their own surface, and exhibit phenomena, such as the dull red heat of iron, or of most kinds of pit-coal, and are therefore more suited for the purpose of producing heat than light. But by using substances which are formed into inflammable vapours, at a temperature below that which is required for the ignition of the substances themselves, gas is obtained and flame is produced. Much light is thus evolved at a comparatively low temperature. The gas necessarily rises above the combustible substance from which it is evolved, owing to its being disengaged at a temperature considerably higher than that of the surrounding air, than which it is necessarily rarer. Of this description are the flames obtained by the burning of the various oils which are generally employed in the illumination of lighthouses. In the combustion of oil, wicks of some fibrous substance, such as cotton, are used, into which the oil ascends by capillary action, and being supplied as required in very thin films, is easily volatilized into vapour or gas by the heat of the burning wick. The gas of pit-coal has been occasionally used in lighthouses, being conveyed in tubes to the burners in the same manner as when employed for domestic purposes. There are certain advantages, more especially in dioptric lights, where there is only one large central flame, which would render the use of gas desirable. The form of the flame,
which is an object of considerable importance, would thus be rendered less variable, and could be more easily regulated, and the inconvenience of the clock work of the lamp which is necessary to maintain the due economy of so large a flame, would be wholly avoided. But it is obvious, that gas is by no means suitable for the majority of lighthouses, their distant situation and generally difficult access rendering the transport of large quantities of coal expensive and uncertain; whilst in many of them there is no means of erecting the apparatus necessary for manufacturing gas. There are other considerations which must induce us to pause before adopting gas as the fuel of lighthouses; for, however much the risk of accident may be diminished in the present day, it still forms a question, which ought not to be hastily decided, how far we should be justified in running even the most remote risk of explosion in establishments such as lighthouses, whose sudden failure might involve consequences of the most fatal description, and whose situation is often such, that their re-establishment must be a work of great time and expense. Gas is, besides, far from being suitable in catoptric lights, to which, in many cases (especially when the frame is moveable, as in revolving lights), it could not be easily applied. The oil most generally employed in the lighthouses of the United Kingdom is the sperm oil of commerce, which is obtained from the South Sea whale (Physeter macrocephalus). In France, the colza oil, which is expressed from the seed of a species of wild cabbage (Brassica oleracea colza), and the olive oil are chiefly used; and a species of the former has lately been successfully introduced into the British light-
houses. Of all these oils, the purified sperm oil has hitherto been generally considered the most advantageous for lighthouse purposes; but the late adoption of the colza oil in many of the British lights, on the suggestion of Mr Joseph Hume, M.P., while chairman of a select committee of the House of Commons on lighthouses, has led to an important saving, as its combustion produces an equal quantity of light at little more than one-half of the expense for spermaceti oil. Considerable experience has justified the following statements made in my Report of 10th March 1847 :
" 1 . The colza oil possesses the advantage of remaining fluid at temperatures which thicken the spermaceti oil so that it requires the application of the frost lamp.
" 2 . It appears, from pretty careful photometrical measurements of various kinds, that the light derived from the colza oil, is, in point of intensity, a little superior to that derived from the spermaceti oil, being in the ratio of 1.056 to 1 .
" 3 . The colza oil burns both in the Fresnel lamp and the single Argand burner with a thick wick during seventeen hours without requiring any coaling of the wick or any adjustment of the damper; and the flame seems to be more steady and freer from flickering than that derived from spermaceti oil.
" 4 . There seems (most probably owing to the greater steadiness of the flame) to be less breakage of glass chimneys with the colza than with the spermaceti oil.
" 5 . The consumption of oil, in so far as that can be ascertained during so short a period of trial, seems in the Fresnel lamp to be 121 for colza, and 114 for sper-
maceti; while in the common Argand, the consumption appears to be 910 for colza, and 902 for spermaceti.
" 6 . If we assume the means of these numbers, 515 for colza, and 508 for spermaceti, as representing the relative expenditure of these oils, and if the price of colza be 3 s . 9 d ., while that of spermaceti is 6 s .9 d . per imperial gallon, we shall have a saving in the ratio of 1 to 1.755 , which, at the present rate of supply for the Northern Lights, would give a saving of about \(£ 3266\) per annum."

Colza oil has been introduced in England into all the lights, whether catoptric or dioptric ; but in Scotland its general use has as yet been confined to the dioptric lights, and such catoptric lights as revolve, and are not likely to be changed to the dioptric system. In the catoptric lights, the only reason for not making an equally extensive trial is the necessity for renewing all the burners, which require to be so constructed as to receive thick wicks of brown cotton; and it has, until lately, been considered prudent to proceed with some caution in changing the apparatus, so as to suit it for burning a patent oil, the circumstances attending the regular and extensive supply and the price of which, can hardly yet be fully known. The change is proceeding gradually from the use of the spermaceti to that of the colza oil ; and, in a few seasons, the whole will be completed, as nearly all the revolving catoptric lights have been altered; and the change on the fixed lights has been only delayed until they shall be converted to the dioptric system, so that one change may serve every purpose at once.

The oxyhydrogen light is produced by the ignition or combustion of a ball of lime ( \({ }_{8}^{3}\) inch diameter) in the united flames of hydrogen and oxygen gases, and is equal to about 264 flames of an ordinary Argand lamp with the best spermaceti oil. It generally bears the name of the late Lieut. Drummond, R. E., who first applied it in the focus of a paraboloïd for geodetical purposes, and afterwards proposed it for lighthouses. (See his Account of the Light, in the Phil. Trans. for 1826, p. 324, and for 1830, p. 383.) The Voltaic light is obtained by passing a stream of Voltaic electricity from a powerful battery between two charcoal points, the distance between which requires great nicety of adjustment, and is the chief circumstance which influences the stability and the permanency of the light. The Voltaic light greatly exceeds the Drummond light in intensity, as ascertained by actual comparison of their effects; but the ratio of their power has not been accurately determined. It was first exhibited in the focus of a reflector by Mr James Gardner (formerly engaged in the Ordnance Survey of Great Britain), whose name, by an equal award, it ought to bear.

Thie application of the oxyhydrogen and electric lights to lighthouse purposes is, owing to their prodigious intensity, a very desirable consummation; but it is surrounded by so many practical difficulties that, in the present state of our knowledge, it may safely be pronounced unattainable. The uncertainty which attends the exhibition of both these lights, is of itself a sufficient reason for coming to this conclusion. But other reasons unhappily are not wanting. The smallness of the flame renders them wholly inapplicable to dioptric instru-
ments, which require a great body of flame in order to produce a degree of divergence sufficient to render the duration of the flash in revolving lights long enough to answer the purpose of the mariner. The same defect of volume completely unfits the combination of these lights with the reflector for the purposes of a fixed light.

In 1835, Mr Gurney proposed the combination of a current of oxygen with the flame of oil, in order to obtain a powerful light of sufficient size to produce the divergence required for the illumination of lighthouses. The Trinity-House of London entertained the proposal, and made some experiments on this important subject ; but the plan was finally rejected as disadvantageous in practice.

Until the invention by Argand (about the year 1784), of the lamp with a double current of air, the art of illumination seems to have received no improvement, and to have occupied very little attention from the time of Cardan, or at all events of Dr Hook, who, about the year 1677, in a monograph entitled "Lampas," made some observations on the constitution of fame, so important as to make one wonder that he should have stopped short of the discoveries of later inventors. Before Argand's time, every wick consisted of a solid cord, whose flame was fed only by the current of air on its outside; and the consequence of this arrangement is, that the stream of vapour or smoke, especially from the centre of thick wicks, escapes unburnt; because, before it reaches the height at which the combustion of the central stream can take place, its temperature has become too low to admit of its ignition.

That the form of a flame is necessarily conoidal, and that its height is determined by the relation subsisting between its diameter and the continually varying velocities of the currents of gas and air, may be easily shewn; and the combustion of each annular film of the stream of gas from the wick can take place only at a level determined by, and continually varying with, the ratio of the velocities of the streams of gas and air. I am unwilling to offer this explanation in my own words, when those of M. Peclet, in his excellent work, Traite de l'Eclairage, are at hand,-" Let us conceive," says he, " a very thin film or layer of inflammable gas placed horizontally, and which rises into the air parallel to itself, with a uniform motion. We shall suppose that it cannot be burnt, except at its circumference, and that the top and bottom of the film are, by some means, preserved from combustion (they are so preserved in ordinary flames, by the films which precede and follow them). If the circumference is at a high enough temperature, it will burn; at each instant the film or layer of air, which has assisted the combustion and also the products of that combustion, being very hot, will rise very rapidly, and will make room for other layers or films of air, which will rise in their turn; and as the diameter of the film of gas is continually diminishing, it is obvious that its combustion will offer the appearance of a series of circles continually growing smaller, and terminating at length in a point. If we trace in thought the series of circles which the combustion has successively developed, we shall form' a cone whose length will depend on the ratio of the velocities of the
films of gas and of air which escape after combustion. If, for example, the velocity of the current of air were very great, compared to the velocity of the cylinder of gas, the entire combustion would take place, while the film of gas passes over a very small space; and the cone, formed by the succession of luminous circles, would consequently be very short. If, on the contrary, there were but a very small difference between these velocities, the luminous circles would only appear at considerable intervals from each other; for the air which had served for combustion, being unable to feed it longer, the surface of the cylinder could not become luminous until the difference of velocity had freed it from the air which had served for the preceding combustion. If, then, we imagine a set of similar films succeeding each other, each of them would give rise to the same series of coloured rings; and as there would be a film in each section of the cone in a state of combustion at the same instant of time, the cone would, of course, appear luminous throughout its height."-Peclet, Traité de EEclairage, p. 51.

The chief improvements which had been made, consisted in varying the level of the oil in the cistern, or in attempts to render that level constant, by mechanical means, and in lessening the thickness of the wick, by spreading its substance into a flat form, thus reducing the stream of gas which escapes from the centre of a thick cylindric wick without being burnt, and thereby causing a more complete combustion, and producing less smoke and a whiter flame. To Argand belongs the great merit of having first formed the wick into a hollow cylinder, thus supplying the flame with two
currents of air, one of which, as in the case of the solid wick, envelopes the flame, and the other, passing through the centre of the wick, is enveloped by the flame itself. He also added a chimney, which served to defend the flame from irregular draughts of air, and to regulate the proportion between the velocities of the currents of air and the stream of gas. This was indeed a most important step in the art of illumination, and causes the great difference between the incomplete combustion, which, owing chiefly, as we have seen, to a defect in the supply of air, always takes place with a solid wick (from which much unburnt gas escapes in the form of smoke), and that more perfect combustion in which passage is given for a free current of air through the centre of the wick. The invention of Argand came nearly perfect from his hands; and but a few slight modifications of his original arrangement have been introduced. The Argand burner consists of two concentric tubes or cylinders, separated by a small annular space, which is shut at the bottom, and communicates by a pipe with the oil fountain, whose level ought to be a little below the level of the upper edge of the cylinders. In this annular space, partly filled with oil from the fountain, stands a cylindric wick of cotton, loosely wove, into which the oil rises freely by capillary action. The wick has its lower edge fixed to a metallic ferule or ring, called a wickholder, which (by means of a peculiar arrangement, to be afterwards described) gives the power of raising or depressing the wick to any convenient level with regard to the burner. A cylinder of glass, of greater diameter than the burner, rests on a gallery or ring which hangs
from the burner and surrounds it. This glass cylinder, or chimney as it is generally called, should stand vertically with its axis coincident with that of the burner itself. The effect of this arrangement is obvious, and has already in part been indicated. The flame is thus necessarily bounded on all sides by two conical concentric surfaces, one external and concave, and the other internal and convex, both of which receive a free current of air. The flame is therefore very thin in every direction; and, as a consequence of the mutual radiation of its different parts on each other, it is throughout its entire surface of more equal temperature than can ever be attained either in the thick solid wick or the narrow flat one. The glass cylinder also increases the force of the two currents which pass outside and inside of the flame; and the union of so many favourable circumstances produces a greater amount of pure light than has yet been obtained by any other method. The contraction of the glass chimney (known by the technical name of the shoulder) at a point a little above the level of the wick, tends to direct the current of air inwards on the flame, thereby causing a more perfect combustion and the evolution of more light.

Great as Argand's improvement undoubtedly was, the value of the lamp alone as a means for the illumination of lighthouses must be regarded as comparatively small. The primary object of a lighthouse is, to give early notice to the mariner of his approach to the coast; and it is therefore necessary that the light be of such a kind that it may be seen at a great distance. Every one is practically acquainted with the fact that
the rays proceed in all directions from a luminous body in straight lines ; and if we could obtain a ball equally luminous in every part of its surface, it would give an equal share of light to every part of the inner surface of a hollow sphere, whose centre shall coincide with the centre of the ball. Again, if an opaque body were placed between the luminous ball and the hollow sphere, the part opposite that body would be deprived of the light by the interception of the rays, and no light would emerge from a hole bored in that part of the surface of the hollow sphere. The bearing of these facts is obvious; and no one can fail to perceive that in the case of a lighthouse illuminated by a single unassisted burner, a seaman could only receive the benefit of that small portion of light which emerges from the lamp in a line joining his eye and the centre of the flame. The other rays would be occupied partly, but in a very small proportion, in making the light visible in other parts of the horizon ; while all the rest would be lost by escaping upwards into the sky, or downwards below the plane in which seamen can see a lighthouse. This state of matters would be little improved by increasing the number of burners, as the effective part of the light would only be augmented by the addition of an equally trifling portion of light from each burner. The small pencils of rays thus meeting at the eye of a distant observer, would form a very minute fraction of the whole quantity of light uselessly escaping above and below the horizon, and also at the back of each flame; and the wasteful expenditure of light would be enormous. By such a method no practically efficient sea-light could ever have been obtained.

\section*{CATOPTRIC* SYSTEM OF LIGHTS.}

For those defects a simple remedy is found in the wellknown power possessed by most bodies, of reflecting or throwing back from them the light which falls upon them. This property is not possessed by all reflecting bodies in an equal degree, some absorbing more, and some less, of the incident light. Perhaps the earliest attempts to apply this property as a corrective for the direction of the rays from a lighthouse, would be confined to placing plane mirrors behind each lamp; yet this would prove but a partial remedy, as it would still leave the greater part of the light to stray above and below the proper direction. Hollow mirrors of a spherical form might next be tried; and if properly placed with reference to the flame, would constitute a very great improvement in lighthouse illumination. But those steps in the march of improvement are more imaginary than real; and I am not aware of any well-authenticated records of such gradual attempts having preceded the adoption of the right mode of applying reflection as a means of rectifying the direction of the rays emerging from a lighthouse. There is, on the contrary, distinct evidence that the impulse given by Argand's invention,

\footnotetext{
* From the Greek xarorrgov, a mirror ; a compound of \(x \alpha r \alpha\),

}
led to an immediate adoption of the most perfect form of reflecting instruments.

The name of the inventor of paraboloidal mirrors, and the date of their first application to lighthouses, have not been accurately ascertained. The earliest notice which I have been able to find, is that by Mr William Hutchinson, the pious and intelligent author of a quarto volume on "Practical Seamanship" (published at Liverpool in 1791), who notices (at p. 93) the erection of the four lights at Bidstone and Hoylake, for the entrance of the Mcrsey, in the year 1763, and describes large paraboloïdal moulds, fashioned of wood and lined with mirror-glass, and smaller ones of polished tinplate, as in use in those lighthouses. Mr Hutchinson seems to have understood the nature, properties, and defects, of the instruments which he describes, and has shewn a good acquaintance with many of the most important circumstances to be attended to in the illumination of lighthouses. Many claims to inventions rest on more slender grounds than might be found in Mr Hutchinson's book for concluding him to have first invented the paraboloidal mirror, and applied it to use in a lighthouse ;* but, in the absence of any statement as to the date when the mirrors were really adopted, the merit of the improvement cannot, in justice, be awarded to him.
M. Teulère, a member of the Royal Corps of Engineers of Bridges and Roads in France, is, by some,
* Mr Hutchinson seems also (" Practical Seamanship," p. 198) to have tried speculum metal as a material for lighthouse reflectors.
considered the first who hinted at the advantages of paraboloidal reflectors; and he is said, in a memoir, dated the 26th June 1783, to have proposed their combination with Argand lamps, ranged on a revolving frame, for the Corduan Lighthouse. Whatever foundation there may be for the claim of \(M\). Teulère, certain it is that this plan was actually carried into effect at Corduan, under the directions of the Chevalier Borda ; and to him is generally awarded the merit of having conceived the idea of applying paraboloìdal mirrors to lighthouses. These were most important steps in the improvement of lighthouses, as not only the power of the lights was thus greatly increased, but the introduction of a revolving frame proved a valuable source of differences in the appearance of lights, and, in this way, has since been the means of greatly extending their utility. The exact date of the change on the light of the Corduan is not known; but as it was made by Lenoir, the same young artist to whom Borda, about the year 1780, entrusted the construction of his reflecting circle, it has been conjectured by some that the improvement of the light was made about the same time. The reflectors were formed of sheet-copper, plated with silver, and had a double ordinate of 31 French inches. It was not long before these improvements were adopted in England, by the TrinityHouse of London, who sent a deputation to France to inquire into their nature. In Scotland, the first work of the Northern Lights Board, in 1787, was to light a lantern on the old castle at Kinnairdhead, in Aberdeenshire, by means of parabolic reflectors and lamps. These reflectors were formed of facets of mirror-glass, placed in
hollow paraboloìdal moulds of plaster, according to the designs of the late Mr Thomas Smith, the engineer of the Lighthouse Board, who (as appears from the article Reflector, in the Supplement to the third edition of the Encyclopædia Britannica) was not aware of what had been done in France, and had himself conceived the idea of this combination. The same system was also adopted in Ireland; and in time, variously modified, it became general wherever lighthouses are known.

To enable us to enter on the subject of the proper forms of reflectors, we must glance very briefly at the laws of reflection. Those laws are two in number. 1st, The ray which falls on a reflecting surface, called the incident ray, and the ray which leaves the reflector, called the reflected ray, are always in one plane, which plane is perpendicular to the reflecting surface. \(2 d\), The angle which the reflected ray makes with the reflector is always equal to the angle which the incident ray makes with it, or, in other words, the angle of incidence is equal to the angle of reflection.*
*This will be more readily understood by referring to the ac-
Fig. 3.

eompanying figure (Fig. 3), in which CDEF is the reflecting sur-

It would lead to prolixity, altogether superfluous in this place, to explain, in a rigorous manner, the effects produced by various reflecting surfaces on the direction of the rays incident on them ; as any one who comprehends the laws of reflection just enumerated, may easily satisfy himself of the following truths: \(1 s t\), That a plane mirror makes no change on the divergence of the rays, but merely causes them to emerge from its surface in the same direction as if they had come from a point as much behind the mirror as the luminous body lies in front of it. \(2 d\), A convex reflecting surface increases divergence, and disperses the rays in the same manner as if they had come directly from a point behind it, whose distance from the mirror increases with the distance of the luminous body from its surface, and diminishes with the degree of convexity of the mirror. \(3 d\), A concave surface diminishes the divergence of the rays incident upon it from a point between the surface and its centre of curvature; the distance of the point in which the reflected rays converge diminishing as the distance of the radiant point, or the concavity of the mirror is increased. It is obvious, therefore, that concave mirrors are those which are required to produce a
face; GHOKI the plane of reflection perpendicular to that surface; BO a line perpendicular or normal to the surface CDEF; and AO the incident ray. Then if in the plane GHOKI, the angle BOI be made equal to \(\mathrm{AOB}, \mathrm{OE}\) is the reflected ray ; BOG is then the angle of incidence; and BOI the angle of reflection. GOH and IOK, which are the complements of those angles, are, indeed, more strictly speaking, the angles of incidence and reflection; but in cases where the reflecting surface is curved, it is more convenient to refer the angles to the normal BO .
correction of the path of the rays, so as to apply them to most advantage in a lighthouse, the object to be attained being that of throwing the greatest amount of light towards given points in the horizon, and collecting the divergent rays, which, as we have already seen, are scattered above and below it.

To simplify our view of this matter, I shall, in the first place, suppose that the object to be attained is, to throw the whole rays of a single lamp, with an infinitely small flame, to a given mathematical point at a moderate distance; and, as this is a case which cannot occur in the practice of lighthouse illumination, I content myself with observing, that this object may be attained approximately by placing the lamp in front of a spherical mirror at any distance greater than half the radius of the curve surface, or accurately by placing it in one focus of an elliptical mirror ; in all those cases, the rays would meet in the opposite, or, as they are termed, conjugate foci. Let us next suppose that our object is to illuminate, by means of a mathematical point of light, a small circular space on the horizon equal in diameter to the mirror employed; this object will be rigorously attained only by placing the light in the focus of a paraboloidal reflector. The same object may be approximately attained by placing the light in a spherical mirror, at a point half-way between the centre of curvature and the surface of the mirror, provided the surface of the mirror shall subtend only a small angle at the centre of curvature. The paraboloidal mirror, on the contrary, has the property of con-
verging to the focus parallel rays falling upon every point of its surface, however extended it may be.

Any one practically acquainted with this subject, must at once perceive that the paraboloïdal mirror completely fulfils one great object required in a lighthouse ; and to render this more obvious to the general reader, I shall, for the present, confine my remarks to the case of those lighthouses which exhibit to the mariner, in every part of the horizon, pencils of light at certain intervals of time, separated by periods of darkness, reserving the consideration of lights which are continually in sight all round the horizon, or over a given portion of \(i t\), for a subsequent part of these observations. In doing this, I am aware that I may appear to be departing from the strict order of investigation, by suddenly introducing the idea of motion; but a little consideration will, I think, satisfy the reader that this is, in reality, the more convenient mode of treating the subject. Let us suppose, then, that our object is to give occasional flashes of light, separated by intervals of darkness, to seamen in various azimuths, and at various distances from a lighthouse. It is obvious that this may be most efficiently done by causing concave mirrors, which collect the rays from lamps placed in them, and thereby increase the light in front of the mirror, to revolve round a vertical axis with a velocity suited to produce the required number of flashes in a given time. The paraboloïdal mirror is best adapted for producing this effect, for the following reasons: \(1 s t\), Because it alone produces a rigorous parallelism of all rays proceeding from its
focus, and falling upon any point of its surface, however distant the point of reflection from that focus, or however far in front of it. 2d, Because it therefore embraces in its action the greatest number of the whole rays coming from the focus, and, cateris paribus, will produce the strongest light. \(3 d\), Because the theoretical object to be attained is to make those flashes equally powerful at any distance, an effect which would be rigorously fulfilled by placing an infinitely small flame in a perfect paraboloìdal mirror. And, \(4 t h\), Because although absolute equality of luminousness at any distance is not attainable, and, in practice, is inconsistent with other conditions required in a useful light, we still, by using the parabolic mirror, make the nearest approach to parallelism of the reflected rays, and consequently obtain the strongest light which is consistent with a due regard to a certain duration of the flash on the eye of a distant observer, which is measured by the angle of the luminous cone projected to the horizon.

Having thus so far anticipated what some might think would more naturally have occurred in a subsequent part of these observations, I return to a more detailed consideration of the parabola itself, and its product, the paraboloidal mirror. I content myself, however, with describing the parabola, by that property which peculiarly adapts it to the purposes of a lighthouse. The parabola, then, is a curve of the second order, obtained by cutting a cone in a plane parallel to one side, and possessing this remarkable property, that a line drawn from the focus to any point in the curve, makes, with a
tangent at that point, an angle equal to that which a line parallel to the axis of the curve makes with that tangent.*
* See third corollary to Proposition III. of Wallace's Conic Sections, which shews that a tangent to the parabola makes equal angles with the diameter which passes through the point of contact and a straight line drawn from that point to the focus. The curve may be traced in two different ways, both dependent on the property, that the distance of any point in the parabola from the focus is equal to its distance from the directrix.

To draw the curve mechanically (fig. 4), let F be the focus, MF the focal distance (chosen at pleasure according to rules which I shall afterwards notice), KMX is the axis, and AB the directrix (the dotted line \(f \mathrm{Fe}\), bounded by the curve at either end, would then be the parameter or latus rectum). Place the edge of the straight ruler AKHB along the directrix; and let

Fig. 4.


LHB be a square ruler which may slide along the fixed ruler AKHB, so that the edge HL may be constantly perpendicular to AB , or parallel to MX, the axis; let LDF be a string equal in length to HL, and having one end fixed in F , and the other at L , a point in the sliding square. Then if the string be stretched

It is easy to see, that if this curve revolve about its axis, it will generate a parabolic conoid, which we may
by a pencil D , so as to keep the part DL close to the edge of the square, and if at the same time the square be gently pushed along the line AB , the point D will be forced to move along the edge LHH of the square, and will trace out a curve which will be the required parabola. This is obvious from the consideration, that the string LDF being equal in length to LH, and LD being oommon to both, the remainder DF must be equal to the remainder DH , so that the point which traces the curve being equidistant from the directrix and the focus must, in terms of the above definition, describe a parabola.

In the second place, the same property, as already stated, furnishes us with the means of tracing the curve by finding successive

Fig. 5.

points therein (fig. 5). Draw a line \(a b\) perpendicular to the axis \(\mathbf{O X}\), and the position in this line, of a point \(p\) through which the curve passes, is easily found, thus: Describe from \(F\) the focus as a centre with a radius equal to the perpendicular distance \(0 d\) of the line \(a b\) from the directrix AB, a circle cutting the line \(a b\) in two points \(p\) and \(p^{\prime}\); then both these points are in the curve. By repeating the same process, any number of points in the curve may be obtained.
conceive to be concave or convex, as we please. If the surface be concave, we obtain the mirror of which we are in search; for every principal section, or that passing through the axis of such a mirror, will necessarily possess the same properties as that of the plane curve, and will each have a focus meeting in one and the same point ; the union of all these sections will therefore form a mirror capable of reflecting, in a direction parallel to the axis and to each other, all the rays of light which fall on its surface.

We have already seen that a perfect paraboloïdal mirror, with a point of light infinitely small placed in the focus, would project a beam equally intense at any distance, every transverse section of which would be of the same superficial extent. In practice, these conditions can never be rigorously fulfilled. No perfect instrument can come from the hands of man; and every mirror must of necessity possess many defects. To obtain a true mathematical point of light is also impossible; and for the purposes of a lighthouse, it would be completely useless, as will appear from the following simple considerations. Let us suppose that a true paraboloïdal mirror, having a double ordinate or space of 2 feet, and illuminated by a point of light, projects a truly cylindric beam of light to the horizon, and that it revolves horizontally round a vertical axis, with such a

Lastly, from the equation to the curve, the length \(y\) of any ordinate may be computed, in terms of \(m\) its principal focal distance, and \(\boldsymbol{x}\) its abscissa, by the simple expression,-
\[
y=\sqrt{4 m x}
\]
velocity as to cause the beam to pass over the eye of an observer stationed at the distance of 100 feet in one second of time, and we shall find that another observer, at a distance of 15 miles from the mirror, would not see the light at all, although of equal size, because its velocity at that distance would be so great as only to be present to his eye for \(\boldsymbol{T}_{7}^{1} \mathrm{~B}^{2} \mathrm{~d}\) part of a second, a space of time far too short to make a perceptible impression on the eye of a distant observer. This is no mere hypothesis unsupported by facts; for I shall have occasion, in another place, to notice certain experiments, by which it was ascertained that a beam of light emerging from a lens, and passing over the eye of an observer at 14 miles' distance, in a space of time equal to \({ }^{\frac{1}{8} \frac{7}{8}}\) th of a second, became altogether invisible at that distance.

For this evil, happily a very simple and efficient re-medy may be found in what may be said to constitute a theoretical defect in the combination of the Argand burner with the reflector. The burner, instead of being a mathematical point, has generally a diameter of about one inch, and a ray proceeding from the edge of the flame to any point on the surface of the mirror, makes, with the line joining that point and the principal focus. an angle which, being repeated by reflection, gives the effective divergence of each side of the mirror at that point.*

\footnotetext{
* This is easily understood by reference to the accompanying diagram (fig. 6), in which \(A O B\) is a central section of a paraboloìdal mirror.
\(\mathbf{P F}=\) distance from the focus F to a point in the curve \(\mathbf{P}\), and PG a tangent drawn from \(P\) to the surface of the flame at \(G\)
}

It is still more obvious, that a perfect paraboloïdal figure, and a luminous point mathematically true, would render the illumination of the whole horizon, by means
\(\mathrm{FG}=\) radius of the wick or flame;
and GPF \(=\mathrm{G}^{\prime} \mathrm{PF}{ }^{\prime}=\) divergence of one side of mirror, and conse-

quently \(2 \mathrm{GPF}=\) the whole effective divergence of the mirror at that cross section.
\[
\text { Now } \sin G P F=\frac{G F}{\mathrm{PF}}
\]
or the "sine of the divergence from each point is equal to Radius of flame.
\(=\) Distance from focus to point of reflection.
It is obvious that this quantity, which varies inversely with the distance of the reflecting surface from the focus, is greatest at the vertex of the curve, and least at the sides or edges of the paraboloid. The most useful part of the light, or that which conduces
of a fixed light, impossible; and it is only from the divergence caused by the size of the flame, which is substituted for the point, that we are enabled to render even revolving lights practically useful. But for this aberration, the slowest revolution in a revolving light would be inconsistent with a continued observable series, such as the practical seamen could follow, and would, as we have seen, render the flashes of a revolving light too transient for any useful purpose; whilst fixed lights, being visible in the azimuths only in which the mirrors are placed, would, over the greater part of the distant horizon, be altogether invisible. The size of the flame.
to the strongest part of the flash in a revolving light, is that which is derived from the cone of rays which is bounded by the limits of this minimum divergence; for the faint light which first reaches the eye of a distant observer, in the revolution of a reflector, is not that which is reflected by the sides or edges, as might at first be supposed, but proceeds from the centre. The light, in fact, gradually increases in power in proportion as additional rays of reflected light are brought to bear on the observer's eye, until, last of all, the extreme edge of the mirror adds its effect. The light continues in its best state until the opposite limit of minimum divergence has been reached, when it begins gradually to decline, receding from the margin of the mirror towards the centre; and, having at length reached the limit of its maximum divergence, it finally disappears at the centre. The increase and decline of the power of a mirror in the course of its movement round the circle of the lantern, as seen by a distant observer, will, therefore, in all its different states, be measured by the areas of a series of circles describad from its focus, with radii equal to the distance of the focus from the point of the mirror which reflects to the observer's eye the extreme ray which can reach him in any given position of the mirror. This will be more easily understood by referring to the accompanying diagram,
therefore, which is placed in the focus of a paraboloidal mirror, when taken in connexion with the form of the (fig. 7), in which ea \(e^{\prime}\) is the principal section of a paraboloidal mirror, F its focus, \(a \mathrm{FA}\) its axis, and FK the radius of the flame. Fig. 7.


If the reflector revolve round a vertical axis at \(O\), an observer placed in front of it (at a distance so great that the subtense of the mirror's width would be small enough to allow us safcly to consider the lines drawn from \(e\) and \(e^{\prime}\) to his eye as parallel), would receive the first ray of light in the direction \(a \mathrm{D}\), as reflected at \(a\), from a single point on the edge of the flame (where a tangent to the flame would pass through \(a\) ); and conversely he would lose the last ray at \(\mathrm{D}^{\prime}\), as reflected at \(a\), from a single point on the opposite margin of the flame; and hence, as above,
mirror itself, leads to those important modifications in the paths of the rays and the form of the resultant beam
the greatest divergence is measured by the angle which the flame subtends at the vertex \(a\) of the mirror, being the sum of the angles \(\alpha\) and \(\alpha^{\prime}\). We shall next suppose the mirror to move a little, so that the observer may receive at \(G\) a ray of light from some other point in the flame which is reflected at \(b\); while another ray from an opposite point reflected at \(b^{\prime}\) would be seen in the parallel direction \(b^{\prime} \mathrm{G}^{\prime}\), thus indicating the boundary of a circular portion of the mirror \(b a b^{\prime}\), the whole of which would reflect light to the distant observer's eye. Again, let us suppose a ray to come from another part of the flame, and be reflected at the mirror's edge \(e\) into the direction \(e \mathrm{H}\), and another from the opposite side of the flame to be reflected at its opposite edge \(e^{\prime}\), into the direction \(e^{\prime} \mathrm{G}^{\prime \prime}\), and we obtain the full effect of the whole reflecting surface, which will continue unabated until the mirror in the course of its revolution shall reflect at \(e^{\prime}\) to the observer's eye, a ray from a point in the margin of the flame (through which a tangent drawn from \(e^{\prime}\) to the flame would pass) in such a direction, that the angle which it makes with the axis of the mirror is equal to that subtended by the radius of the flame at the distance Fe or Fe \(e^{\prime}\). After this the light would recede from the edges of the mirror in the same gradual manner, until it should vanish in the direction \(a \mathrm{D}^{\prime}\), which is the opposite limit of the extreme divergence of the instrument. In the above explanation, I have confined myself simply to the effects of the outer ring of the flame, which is the source of divergence; but I need not remind the reader that every portion of the flame radiates light, which, being reflected, conduces to the effect. Some rays also are passing from the opposite sides of the flame through the true focus, so as to be normally reflected in lines parallel to its axis. The solid lines in the diagram shew the theoretical reflection of rays proceeding from F to \(b, b^{\prime}, e, e^{\prime}\), where they are diverted into the directions \(b \mathrm{~B}, b^{\prime} \mathrm{B}^{\prime}, e \mathrm{E}\), and \(e^{\prime} \mathrm{E}^{\prime}\); and by contrast with the dotted lines, serve to render more perceptible the path of the divergent rays which come from the edge of the flame. The Greek letters indicate the angles of divergence, and point out
of light, which have rendered the catoptric system of lights so great a benefit to the benighted seamen.

In order to obtain a mirror capable of producing a given divergence of the reflected beam, therefore, we must proportion its focal distance to the diameter of the flame in such a manner, that the sine of one-half of the whole effective divergence of the mirror, may be equal to the quotient of the radius of the flame, divided by the distance of a given point on the surface of the mirror from the focus. The best proportions for paraboloidal mirrors depend on the objects which they are meant to attain. Those which are intended to give great divergence to the resultant beams, as in fixed lights, capable of illuminating the whole horizon at one time, should have a short focal distance; while those mirrors which are designed to produce a nearer approach to parallelism (as in the case of revolving lights, which illuminate but a few degrees of the horizon at any one instant of time), will have the opposite form. Those two objects may, no doubt, be attained with the same mirror, by increasing or diminishing the size of the burner; but that is by no means desirable, as any change on the size of a burner, which is found to be the best in other respects, must be considered as to some extent disadvantageous.

What I have stated above as to the use of mirrors their relations to each other on either side of the mirror. The arcs of greatest and least divergence are marked in the diagram. This subject will be found treated less directly, but, certainly, more concisely and neatly, by Mr W. H. Barlow, in a paper on the Illumination of Lighthouses, in the London Transactions for 1837, p. 218.
with a short focal distance for lights of great divergence, proceeds on the assumption, that the penumbral portion of the light on each side of the strongest beam (which is confined within the limits of the least divergence, due to that portion of the mirror where the focal distance is the greatest) is to be pressed into service in the illumination of the horizon; and it is the chief inconvenience which attends the application of paraboloiidal mirrors to fixed lights, that because it is impracticable to apply a number of mirrors sufficient to light the whole horizon with an equally strong light, spaces occur on either side of each reflector in which the mariner has a light sensibly inferior to that which illuminates the sector near the axis of each mirror. This will be best explained by stating the numerical results of the computations of the divergence of the mirrors used in the Northern Lighthouses for this purpose, both at the vertex and the sides. In a mirror whose focal distance is 4 inches, and its greatest double ordinate 21 inches, illuminated by a flame 1 inch in diameter, we find by computation, that the greatest divergence is \(14^{\circ}\) \(22^{\prime}\), and that the strongest are of light is only \(5^{\circ} 16^{\prime}\); a difference so great, that while the one may admit of the horizon being imperfectly illuminated by means of 26 reflectors, the superior light which would result from confining the duty of each instrument within the range of its best effect, could only be obtained by the use of 68 reflectors, and the expenditure of a proportionately great quantity of oil, not to speak of the great practical difficulty which would attend the arrangement of so many lamps in a lantern of moderate size. In revolv-
ing lights, the mirrors are not, as in fixed lights, inconveniently taxed for horizontal divergence ; because each portion of the divergent beam visits successively each point of the horizon. In this view of the merits of fixed and revolving lights, I should be disposed to recommend, in any new organisation of lights with parabolic reflectors, the adoption, in fixed lights, of reflectors with a short focal distance and small span, so as to admit of many being ranged around the frame; while in revolving lights, it would be my aim to approach the largest size of reflector that could be made, so as, if possible, to illuminate each face of the revolving frame by means of a large lamp in a single mirror, with a great focal distance, thereby diminishing the difference between the divergence of the powerful cone of rays reflected from the more distant parts of the mirror, and that of the foebler and more diffuse light from its apex. The application of reflectors of very short focal distance, to the flames of the ordinary Argand lamp, must be avoided, as an undue aberration of the resultant cone of light is thereby produced. The difficulty of making the retlectors, also, increases rapidly with their depth.

The maximum luminous effect of the reflectors ordinarily employed in fixed lights, as determined by observation, is generally equal to about 350 times the effect of the unassisted flame which is placed in the focus; while for those employed in revolving lights, which are of larger size, it is valued at 450 . This estimate, however, is strictly applicable only at the distances at which the observations have been made, as the proportional value of the reflected beam must necessarily vary with
the distance of the observer, agreeably to some law dependent upon the unequal distribution of the light in the illuminous cone which proceeds from it. The effect also varies very much in particular instruments. The ordinary burners used in lighthouses are one inch in diameter, and the focal distance generally adopted is 4 inches; so that the extreme divergence of the mirror in the horizontal plane may be estimated at about \(14^{\circ} 22^{\prime}\); while the divergence of the most luminous cone is \(5^{\circ} 16^{\prime}\) for the small reflectors, and \(4^{\circ} 25^{\prime}\) for the larger size. In arranging reflectors on the frame of a fixed light, however, it is advisable to calculate upon a less amount of effective divergence, for beyond \(11^{\circ}\) the light is very feeble; but the difficulty of placing many mirrors on one frame, and the great expense of oil required for so many lamps, have generally led to the adoption of the first valuation of the effective divergence.

The measure of the illuminating power of a paraboloidal mirror may be estimated as the quotient of the sURFace of the circle which cuts it in the plane of its greatest double ordinate, divided by the surface of the largest vertical section of the flame, and diminished by the loss of light in the process of reflection. This estimate will be found near enough for all practical purposes; but it is obviously inaccurate, inasmuch as it overlooks the circumstance of the focal distance of each portion of the mirror being different, and the consequent increase in the length of the various trajectories at each point of the surface as you recede from the axis; and the only correct rule, therefore, is, to find an imaginary focal distance which must be the radius of a
spherical segment, which shall answer the double condition of having its surface equal to that of the greatest cross section of the mirror, and of including, at the same time, a number of degrees equal to those which are brought under the influence of the reflecting action of the paraboloid. This subject, however, as I have already hinted, is not of great practical importance; and I shall not therefore dilate on it farther, but content myself with saying, that such a line will be found to be a mean proportional between the greatest and least focal distances of the mirror. The large mirrors used in the Northern Lighthouses have about \(\frac{1}{1} \frac{2}{7}\) ths of the whole light of the lamp incident on their surface; the rest escapes in the comparatively useless state of naturally radiating light. Several arrangements have been proposed for economising this light, which will be afterwards noticed.

The reflectors used in the best lighthouses are made of sheet-copper, plated in the proportion of six ounces of silver to sixteen ounces of copper. They are moulded to the paraboloïdal form, by a delicate and laborious process of beating with mallets and hammers of various forms and materials, and are frequently tested during the operation by the application of a mould carefully formed. After being brought to the curve, they are stiffened round the edge by means of a strong bizzle, and a strap of brass which is attached to it for the purpose of preventing any accidental alteration of the figure of the reflector. Polishing powders are then applied, and the instrument receives its last finish.

Two gauges of brass are employed to test the form of
the reflector. One is for the back, and is used by the workmen during the process of hammering, and the other is applied to the concave face as a test, while the mirror is receiving its final polish. It is then tested, by trying a burner in the focus, and measuring the intensity of the light at various points of the reflected conical beam. Another test may also be applied successively to various points in the surface, by masking the rest of the mirror; but as it proceeds upon the assumption that the surface of the reflector is perfect, and that we can measure accurately the distance from a radiant coincident with the focus to the point of the mirror to be tried, it is in practice almost useless. For such a trial we must place a screen in the line of the axis of the mirror at some given distance from it, and ascertain whether the image of a very small object placed in the conjugate focus, which is due to the distance of the screen in front of the focus, be reflected to any point considerably distant from the centre of the screen through which the prolongation of the axis of the mirror should pass. We thus obtain a measure of the error of the instrument. For this purpose, we must find the position of the conjugate focus, which corresponds to the distance of the screen. If \(b\) be the distance to which the object should be removed outwards from the principal focus of the mirror, \(d\) the distance from the focus to the screen, and \(r\) the distance from the focus to that point of the mirror which is to be tested, we shall have \(b=\frac{r^{2}}{d}\) as the distance to which the object must be
removed outwards from the true focus on the line of the axis.*

The flame generally used in reflectors is from an Argand fountain-lamp, whose wick is an inch in diameter. Much care is bestowed upon the manufacture of the lamps for the Northern Lighthouses, which sometimes have their burners tipped with silver to prevent wasting by the great heat which is evolved. The burners are

\footnotetext{
* The truth of this equation may be easily ascertained as follows (See fig. 8) :-
}
\[
\text { Fig. } 8 .
\]


Let AP be the mirror, F its principal focus, and PH the line of reflection of the ray FP; then an object at I will be reflected at \(P\) to the conjugate focus \(O\), where the screen is supposed to be placed. But by construction, \(\mathrm{FPI}=\mathrm{HPO}=\mathrm{POF}\), and the angle at F being common, the triangle FPI is similar to FPO, and hence \(\mathrm{FO}: \mathrm{PF}:: \mathrm{PF}: \mathrm{FI}\), and \(\mathrm{FI}=\frac{\mathrm{PF}^{2}}{\mathrm{FO}}\); and substituting the letters in the text, we get \(d: r:: r: b\), and \(b=\frac{r^{2}}{d}\).
also fitted with a sliding apparatus, accurately formed, by which they may be removed from the interior of the mirror at the time of cleaning them, and returned exactly to the same place, and locked by means of a key; it is remarkable that this arrangement should not have been adopted elsewhere, except, indeed, in Ireland, where eight of the lighthouses were many years since fitted up with apparatus made at Edinburgh, under the directions of Mr Robert Stevenson. This arrangement, which is shewn in figs. 9,10 , and 11 , is very important, as it insures the burner always being in the focus, and does not require that the reflector be lifted out of its place every time it is cleaned; so that, when once carefully set and screwed down to the frame, it is never altered. In these figs. a a a represents one of the reflectors, \(b\) is the burner, and \(c\) a cylindric fountain, which contains 24

Fig. 9.
 ounces of oil. The oil-pipe, the fountain \(c\) for supplying oil, and the burner \(b\), are connected with the rectangular frame \(d\), which is moveable in a vertical direction upon the guide-rods \(e\) and \(f\), by which it can be let down, so that the burner may be lowered out of the reflector, by simply turning the handle \(g\) (as will be more fully understood by examining figs. 9 and 10 ), which has the effect of forcing a thread (like that of a screw) on the outside of the guide into a groove in the frame, or withdrawing it, and thus allows it to slide down or locks it at plea-
sure. An aperture of an elliptical form, measuring about two inches by three, is cut in the upper and lower part of the reflector, the lower serving for the free egress and

ingress of the burner, and the upper, to which the cop-per-tube \(h\) is attached, serving for ventilation ; \(i\) shews a cross section and a back view of the main bar of the chandelier or frame on which the reflectors are ranged, each being made to rest on knobs of brass, one of which, as seen at \(k k\), is soldered to the brass band \(l\), that clasps the exterior of the reflector. Fig. 9. is a section of the reflector \(a a\), shewing the position of the burner \(b\), with the glass chimney \(b^{\prime}\), and the oil-cup \(l\), which receives any oil that may drop from the lamp. Fig. 11 shews the apparatus for moving the lamp up and down, so as to remove it from the reflector at the time of cleaning it. In the diagram (fig. 11) the fountain \(c\) is moved partly
down; \(d d\) shews the rectangular frame on which the burner is mounted, \(e e\) the elongated socket-guides through which the guide-rods slide, and \(f\) the guiderod, connected with the perforated sockets on which the checking-handle \(g\) slides. The oil-cup \(l\) (covered with a lid and wick-holder, as shewn in fig. 12) also serves as a frost-lamp during the long nights of winter, when the oil is apt to turn thick. It is attached to the lower part of the oil-tube by the arm \(h\); and is lighted about

Fig. 12.
 an hour before sunset, so as to prepare the reflector lamp for lighting at the proper time. The communication between the burner and the fountain is easily opened or shut in the burners used in the Scotch lighthouses, by simply giving the fountain a turn of one quadrant of the horizon round its own vertical axis by means of the round knob at its top, and thereby moving a simple slide-valve, which shuts off the communication between the fountain-tube and the lamp-tube. By this mode, the oil is cut off about fifteen minutes before extinguishing the lights, so that when that is done, the burner is quite free of oil.

It would needlessly occupy much time and space to describe the various means (many of them sufficiently clumsy) which have been employed, and in many places are still in use, for raising and depressing the wick; it will be enough to say, that they all involve some application of the rack and pinion. I shall, therefore, only describe the method (invented, it is believed, by M. Verzy) which is adopted throughout the district of the Commissioners of Northern Lighthouses. The arrange-
ment is as follows (see figs. 13, 14, 15, 16, 17): The inner tube \(t\) of the burner is enclosed by a strong tube

Fig. 13.


Fig. 14.


S, which fits to it tightly, so as not to be easily moved. This strong tube has a spiral groove cut on its outer or convex surface. The wick-holder has two small pegs projecting from it, the one on the inside (not seen), and the other on the outside at \(a\) (fig. 14). That on the inside works in the spiral groove of the tube S (figs. 13 and 14), already described as embracing the inner tube \(t\); and all that is required for raising the wick is, to make the wick-holder turn round on its vertical axis. This is effected by means of the small external peg \(a\) of the wick-holder (fig. 14), which moves in a vertical slit
\(a\) (figs. 13 and 15), cut in a tube standing in the burner, and concentric with it, and which also moves freely

Fig. 15.


Fig. 16.


Fig. 17.

round its axis. Small knobs \(n n\) (figs. 13, 15, and 17), at the top of this tube, fit into a notch in the upper ring of the gallery, which supports the glass chimney. By turning this gallery \(g\) (see figs. 13 and 17), therefore, motion is given to the tube, with its knobs \(n n\), whose vertical slit \(a\) (while it holds the external peg of the wick-holder, and also turns it round along with it) permits that peg \(a\) to slide upwards or downwards, and thus the wick-holder rises or falls, according as its own internal peg moves up or down the spiral groove in the tube S . In fig. 13, C shews the glass chimney resting on the gallery \(g g\). The wick is shewn in fig. 16, attached to the wick-holder.

An important point in the economy of the Argand lamp, is the level at which the outlet for the oil, in its passage from the fountain to the burner, should be cut.

The cutting of this hole (generally called the flow-
hole) in the pipe is termed the flowing of the lamp, and is commonly done by successive trials, until the oil stands in the burner at the proper level, before the wick is put in. A more ready and accurate method of accomplishing this object and at once determining the level at which the flow-hole should be cut, was introduced by Mr James Murdoch, the Foreman of lightroom repairs to the Scotch Board, and is generally employed in the Northern Lighthouses. Its nature will be readily understood by a reference to the accompanying diagram (fig. 18).

Fig. 18.


The hatched surface represents a metallic ruler, with a spirit-level at L ; C is the cup in which the bottom of the fountain \(f\) (shewn in dotted lines) rests. When the fountain is removed, and the ruler rests on the edge of the \(\operatorname{cup} \mathrm{C}\), the screw at A is used to adjust the level at L ; and a gauge \(\mathrm{G} G\) is allowed to fall until a notch in it at \(x^{\prime}\) rests on the outer tube of the burner \(\mathbf{F}\); the pinching-screw B retains this ruler in its place, and the point \(x^{\prime}\) indicates the level at which the oil should stand in the burner. The level line \(a^{\prime} x\) indicates the level on which the top of the flow-hole H should be cut in the fountain-tube, which is shewn in dotted lines within the outer tube or body of the lamp. In other words, \(y^{\prime} x^{\prime}\) measures the level at which the oil should stand in the burner below the lower edge of the metallic ruler, while the corresponding line \(y x\), at the opposite end, shews the level of the top of the fow-hole H , below the edge of the cup C. The gauge GG applied to that point of the fountain which coincides with the edge of the cup (so that \(y^{\prime}\) coincides with \(y\) ) measures the length \(y x=y^{\prime} x^{\prime}\); and a set-square applied at \(x\) gives the position of H on the fountain-tube. The round dot at \(a\) shews the position of the air-hole in the body of the lamp, which establishes a connection between the external air and the surface of the oil. The rods \(\mathrm{SS}^{\prime}\) shew the sliding gear (described as \(d\) and \(f\), pages 95 and 96 ), and are only introduced to identify this diagram with those of the fountain and burner which have preceded it.

The most advantageous level of the flow-hole depends on many circumstances too obscure and complicated to admit of any systematic elucidation; and it is enough for
all practical purposes, to know that the capillary powers of the wick, and the greater or less viscidity of the oil are the chief circumstances which determine that level. Actual experience is the only sure guide to the best practice in this respect ; and I therefore content myself with stating, that it is generally found that the sperm oil should stand in the empty burner at about \(\frac{8}{8}\) inch below its top. For colza oil \(\frac{8}{g}\) inch is sufficient. In summer, owing to the oil being more fluid, there is sometimes a tendency to overflow the burner; but any inconvenience arising from it is avoided by the plan adopted in the Northern Lights, of shutting off the oil (by means of the apparatus already alluded to on p. 97) about fifteen minutes before extinguishing the lights in the morning.

The arrangement for cutting of the oil is very simple, as will be seen from the annexed diagram (fig. 19), in which F is the fountain, T the oil-tube leading to the burner, and V the flowhole, with its sliding valve. By turning the handle H one quadrant of the circle, the whole fountain F and tube T turn round their vertical axis, while the valve V , which rests in a notch in the cup of the lamp, remains still, and sliding over T, opens the fow-hole. S is the screwplug which retains the oil in the fountain, and which is unscrewed and removed when the fountain is to be filled.

In the reflecting apparatus of the Northern Lighthouses, the focal position of the lamp is not, as we have already seen, liable to derangement, by the re-

moval of the burner for the purpose of cleaning, as the sliding-gear described at p. 95 insures the return of the lamp to its true place. The burner is originally set by means of a gauge, which touches four points of the mirror's surface (one of them being its vertex, and the other three in the vertical plane of its greatest double ordinate). This gauge being provided with a short tube or collar properly placed for the purpose of receiving the burner, at once verifies its true position, both vertical and horizontal. The diagrams 20 and 21 shew the nature of the apparatus for adjusting the burners, the one being a plan and the other a section. The four points which touch the curve are one \(g\) at the vertex, two in the same horizontal plane with

Fig. 20.

the focus, and near the edge of the mirror at PP, and the fourth, also near the edge, and in the same vertical

Fig. 21.

plane with the focus. F is the focus. The horizontal arms are graduated, and fitted with sliding pieces and clamping screws at \(R\), so as to admit of being varied with the width of the mirror ; but each gauge applies only to curves of the same focal distance ; the distance \(\mathrm{F} g\) being fixed. The gauge, when applied to the mirror, is properly secured by the screws at \(R, R\), and \(R^{\prime}\); and the burner which is attached to the oil-tube in a temporary manner at A , is raised into the interior of the mirror. If the tube of the burner ascends into the
circular tube at F until (when fixed by the checking handle already noticed at p. 95) its upper edge just touches a narrow projection inside the tube \(\mathbf{F}\) (so placed that the rim of the burner should simply touch it when it is on the level required for putting the brightest part of the flame in the focus), then the burner is in the proper position ; but if, on the one hand, the axis of the burner stands beyond F , at some point between it and N (which lies in the plane of the mirror's edge), the bent tube O from the fountain must be shortened at A; and if it rise too high, that tube must be bent down (and vice versa), until, by successive trials, it shall exactly fit into the tube F, and stand at the proper level. A skilful workman soon comes to guess those quantities very accurately; and, almost at the first trial, curtails the tube to the proper length, and bends it to the suitable level. All that is needful is, to proceed cautiously, so as not to cut the tube too short, for this leads to some trouble.

The great advantage derived by seamen from the establishment of lights on a coast, soon makes the calls for additional lights so frequent, that their very number itself produces a new evil, in the difficulty of distinguishing the lights from each other. As the object of a light is to make known to the benighted mariner the land he has made, with as much certainty as the sight of a hill or tower would shew him his position during the day, it becomes an object of the first importance to impress upon each light a distinctive character, which shall effectually prevent the possibility of its being mistaken for any other.

Catoptric lights are susceptible of nine separate disE 2
tinctions which are called fixed, revolving white, revolving red and white, revolving red with two whites, revolving white with two reds, flashing, intermittent, double fixed lights, and double revolving white lights. The first exhibits a steady and uniform appearance, which is not subject to any change; and the reflectors used for it (as already noticed) are of smaller dimensions than those employed in revolving lights. This is necessary, in order to permit them to be ranged round the circular frame, with their axes inclined at such an angle, as shall enable them to illuminate every point of the horizon. The revolving light is produced by the revolution of a frame with three or four sides, having reflectors of a larger size grouped on each side, with their axes parallel; and as the revolution exhibits ouce in two minutes, or once in a minute, as may be required, a light gradually increasing to full strength, and in the same gradual manner decreasing to total darkness, its appearance is extremely well marked. The succession of red and white lights is caused by the revolution of a frame whose different sides present red and white lights; and these, as already mentioned, afford three separate distinctions, namely, alternate red and white ; the succession of two white lights after one red, and the succession of two red lights after one white light. The flashing light is produced in the same manner as the revolving light; but owing to a different construction of the frame, the reflectors on each of eight sides are arranged with their rims or faces in one vertical plane, and their axes in a line inclined to the perpendicular, a disposition of the mirrors which, together with the greater quickness
of the revolution, which shews a flash once in five seconds of time, produces a very striking effect, totally different from that of a revolving light, and presenting the appearance of the flash alternately rising and sinking. The brightest and darkest periods being but momentary, this light is farther characterised by a rapid succession of bright flashes, from which it gets its name. The intermittent light is distinguished by bursting suddenly into view and continuing steady for a short time, after which it is suddenly eclipsed for half a minute. Its striking appearance is produced by the perpendicular motion of circular shades in front of the reflectors, by which the light is alternately hid and displayed. This distinction, as well as that called the flashing light, is peculiar to the Scotch coast, having been first introduced by the late engineer of the Northern Lights Board. The double lights (which are seldom used except where there is a necessity for a leading line, as a guide for taking some channel or avoiding some danger) are generally exhibited from two towers, one of which is higher than the other. At the Calf of Man, a striking variety has been introduced into the character of leading lights, by substituting for two fixed lights, two lights which revolve in the same periods, and exhibit their flashes at the same instant ; and these lights are, of course, susceptible of the other variety enumerated above, that of two revolving red and white lights, or flashing lights, coming into view at equal intervals of time. The utility of all these distinctions is to be valued with reference to their property of at once striking the eye of an observer and being instantaneously obvious to strangers.

The introduction of colour, as a source of distinction, is necessary, in order to obtain a sufficient number of distinctions; but it is in itself an evil of no small magnitude ; as the effect is produced by interposing coloured media between the burner and the observer's eye, and much light is thus lost by the absorption of those rays, which are held back in order to cause the appearance which is desired. Trial has been made of various colours; but red, blue, and green alone have been found useful, and the two latter only at distances so short as to render them altogether unfit for sea-lights. Owing to the depth of tint which is required to produce a marked effect, the red shades generally used absorb from ths to \(\frac{5}{6}\) ths of the whole light, an enormous loss, and sufficient to discourage the adoption of that mode of distinction in every situation where it can possibly be avoided. The red glass used in France absorbs only \%ths of the light; but its colour produces, as might be expected, a much less marked distinction to the seaman's eye. In the lighthouses of Scotland, a simple and convenient arrangement exists for colouring the lights, which consists in using chimneys of red glass, instead of placing large discs in front of the reflectors.

After what has been already said on the subject of divergence, it will at once be seen, that in revolving lights the reflectors are placed with their axes parallel to each other, so as to concentrate their power in one direction ; whilst in fixed lights it is necessary, in order to approach as near as possible to an equal distribution of the light over the horizon, to place the reflectors, with their axes inclined to each other,
at an angle somewhat less than that of the divergence of the reflected cone. For this purpose, a brass gauge (see fig. 22), composed of two long arms, AM, AM,

Fig. 22.

somewhat in the form of a pair of common dividers, connected by means of a graduated limb A, is employed. The arms having been first placed at the angle, which is supplemental to that of the inclination of the axes of the two adjacent mirrors at \(O\), are made to span the face of the reflectors, one of which is moved about till its edges are in close contact with the flat surface of one. of the arms of the gauge.

Figs. 23 and 24 shew an elevation and plan of a revolving apparatus on the catoptric principle. In these
figures, \(n n\) shews the reflector flame or chandelier; oo, the reflectors with their oil-fountains \(p p\). The whole

is attached to the revolving axis or shaft \(q\). The copper tubes \(r r\) convey the smoke from the lamps; \(s s\) are cross bars which support the shaft at \(t t\); \(u u\) is a copper pan for receiving any moisture which may accidentally enter at the central ventilator in the roof of the lightroom; \(l\) is

Fig. 24.

a cast-iron bracket, supporting the cup in which the pivot of the shaft turns; \(m m\) are bevelled wheels, which convey motion from the machine to the shaft. The machinery does not require any particular notice, being that of common clock work, moved by the descent of a weight.

Fig. 25 shews a plan of one tier of reflectors arranged Fig. 25.

in the manner employed in a fixed catoptric light; \(n n\) shews the chandelier, \(q\) the fixed shaft in the centre, which supports the whole, oo the reflectors, and \(p p\) the fountains of their lamps. In this figure (in order to prevent confusion) only one tier of reflectors is shewn; the other tiers are so arranged, that their axes divide into equal angles the arcs intercepted between the axes of the adjoining reflectors on the first tier, thereby producing the nearest approach to an equal distribution of the light, which is attainable by this arrangement.

In lighthouses of moderate height, the proper position for the reflector itself is perfect horizontality of its axis, which may be ascertained with sufficient accuracy, by trying with a plummet, whether the lips of the instrument, which we may conclude to be at right angles to the plane of its axis, be truly vertical. In lightrooms very much elevated above the sea, however, the dip of the horizon becomes notable; and a slight inclination forwards should be given to the face of the reflectors, so that their axes produced may be tangents to the earth at the visible horizon of the lightroom ; an arrangement which, in practice, may be easily made by reflecting the sea horizon, in a small mirror placed at the focus, and inclined at \(45^{\circ}\) to the axis of the paraboloid, so that the image of the sea-line may reach the eye in the line of the parameter, in the same manner, as is afterwards noticed, in speaking of the inclination of the curved mirrors used in addition to the refractors in certain dioptric fixed lights. This dip of the reflector, however, must not be permitted to interfere with the perfect horizontality of the top of the burner, which is indispensable to its proper burning.

Various forms of the parabolic mirror were invented by M. Bordier Marcet, the pupil and successor of Argand, who has laboured with much enthusiasm in perfecting catoptric instruments, more especially with a view to their application in the illumination of lighthouses and the streets of towns. Amongst many other ingenious combinations, he has invented and constructed an apparatus which is much used in harbour-lights on the French coast, where it is known by the fanciful name of Fanal* sidéral. The object is to fulfil, as economically as possible, the condition required in a fixed light, of illuminating, with perfect equality, every part of the horizon, by means of a single burner ; and M. Bordier Marcet has, in his workshop at Paris, an instrument of this kind, eight feet in diameter, which he constructed on speculation. The apparatus used in harbour-lights, on the French coast, is of much smaller dimensions, and does not exceed fifteen inches in diameter. A perfect idea of the construction and effect of this instrument may be formed, by conceiving a parabola to revolve about its parameter as a vertical axis, so that its upper and lower limbs would become the generating lines of two surfaces possessing the property of reflecting, in lines parallel to the axis of the parabola, all the rays incident upon them, from a light placed in the point where the parameter and axis of the generating parabola intersect each other. This point being the focus of each parabolic section of this apparatus, light is equally dispersed in every point of the horizon, when

\footnotetext{
* Fanal, from ¢avov, a lantern.
}
the axis of the parabolic section is in a plane perpendicular to a vertical line. But however perfectly this apparatus may attain its important object, it necessarily produces a feeble effect; because, as its action is entirely confined to the vertical direction, the light distributed by it decreases directly as the distance of the observer. This beautiful little instrument is shewn at fig. 26 , in which \(b\) shews the burner, \(p p\) the upper reflecting surface, and \(p^{\prime} p^{\prime}\) the lower reflecting surface, both generated in the manner above described by the revolu-

Fig. 26.

tion of a parabola about its parameter \(\boldsymbol{x b} \boldsymbol{b} ; \mathrm{F}\) is the focus of the generating parabola; and \(l l\) are small pillars which connect the two reflecting plates, and give strength to the apparatus.
M. Bordier Marcet has also prepared an ingenious modification of the paraboloidal mirror, which he has described under the name of fanal à double effet; and the object of which is to obtain a convenient degree of divergence from parabolic mirrors, by the use of two flames and two reflecting surfaces, each of which is
acted upon by its own flame, and also by that of the other. This modification consists in the union of two portions of hollow paraboloidal mirrors, generated by the revolution of two parabolas about a common horizontal axis, and illuminated by two lamps placed in the focus of each. The first surface is generated by the revolution on its axis of a segment of a paraboloid intercepted between the parameter and some double ordinate greater than it, and may, from its form, be called the ribbon-shaped mirror. The second surface is that of a parabolic conoid, which is cut off by a vertical plane passing through a double ordinate, which is equal to the parameter of the parabolic ribbon, which is placed in front of it. The elements of the curve which forms the conoïdal mirror, must be so chosen as to have its focus at a convenient distance in front of that of the ribbonshaped mirror, so as to admit of placing the two lamps separate from each other, as well as to produce the necessary degree of divergence, which is to be obtained by the action of these mirrors respectively on the flame placed in the focus of the other. These two mirrors are joined together in the line of the parametric section of the ribbon, which coincides with the lips of the conoid at some double ordinate behind its parameter. Each mirror produces, by means of the lamp placed in its focus, an approach to parallelism of the reflected rays, which the designer has not inaptly termed the principal effect; whilst the action of each surface on the lamp which is placed in the focus of the other, causes what the inventor calls the secondary or lateral effect. Their secondary action may be described thus: The
lamp, which is in the focus of the ribbon, is much nearer the vertex of the conoid than its own focus; so that its rays making, with normals to the surface of the conoid, angles greater than those which are formed by the rays proceeding from its focus, are of necessity reflected in lines diverging from the axis of the mirror. Those, on the contrary, which proceed from the focus of the conoid, meet the ribbon-shaped surface, so as to make angles with its normals more acute than those which the rays from its own focus could do, and which are, therefore, reflected in lines converging to the axis of the mirror. Those reflected rays must therefore cut the axis, and diverge from it on the other side. This apparatus has been used at La Hève, near Le Havre, and some other lights on the French coast; but it is impossible not to perceive the great loss of light which results from the use of two flames in one mirror ; and it must not be forgotten, that the divergence which is obtained by means of it is not confined to the horizontal direction in which only it is wanted; but that the light is at the same time scattered in every direction round the edge of the mirror.

Arrangements of a similar kind were proposed and executed for the same purpose of uniting greater divergence with considerable power in the central parts of the resultant beam, by Argand himself, in 1806, and also in 1808, by M. Haudry, Ingénieur des Ponts et Chaussées. Argand proposed the union of a paraboloid and an ellipsoid, having their foci coincident in one point, which, being the posterior focus of the latter curve, was illuminated by the rays reflected to it by
means of the ellipsoidal surface from the lamp placed in the anterior focus. From the optical foous thus obtained, some rays would fall on the paraboloidal surface and produce, by reflection, a cylinder of parallel rays, while the rest would diverge from the axis, and form a zone of spreading rays. M. Haudry's plan consisted of a combination of a conical with a paraboloidal mirror so placed, that the rays from the front part of the hollow cone might be nearly parallel to those sent out by the paraboloid; while the rays from its base, diverging from the axis, might produce a ring of divergent rays, similar to that obtained from the ellipsoid of Argand's apparatus. It would occupy much time to exhibit all the disadvantages of the arrangements in the fanal à double effet, and also in those of Argand and Haudry ; and I shall therefore dismiss the subject by observing that the loss of light due to the position of the flame in the apparatus of Argand, is so great as to induce one to wonder that such combinations should ever have been attempted. There can be no doubt, that the most efficient mode of obtaining due divergence from mirrors is, to adopt the paraboloid, with a short focal distance, which has the double advantage of increasing the divergence which is due inversely to the focal distance, and, at the same time, subjecting to the action of the mirror a larger portion of the luminous sphere proceeding from the flame. This, however, as already noticed, may be pushed too far.

Lastly, I shall notice M. Bordier Marcet's fanal à double face, which consists of two paraboloidal mirrors, truncated in the verticat plane of the parameter, and
united together back to back, so as to be illuminated by the same lamp placed in their common focus. To save the light which would otherwise escape the catoptric action, he adds a parabolic conoild of greater focal distance and so placed, that while its focus may coincide with the common focus of the other mirrors, its size may be so restricted, that it shall not interfere with the effect of the truncated mirror opposite which it is situate. The obvious consequence of such an arrangement is, that the rays (see fig. 27) produced from a lamp in the common focus of the three mirrors, will produce in opposite directions a luminous ring from each of the truncated mirrors \(A C, B C\), and \(A^{\prime} C^{\prime}, B^{\prime} C^{\prime}\), while the

Fig. 27.

central or conoildal mirror MN will fill the interior of one of those luminous rings with a cone of rays, whose intensity will be in the inverse ratio of \(\mathrm{MN}^{2}\) to \(a b^{2}\) (or \(\mathrm{FM}^{2}\) to \(\mathrm{F} a^{2}\) ), which latter surface represents the whole amount of naturally divergent rays, which strike on \(a b\), and which are spread over MN. Two sets of reflectors of this form facing in opposite directions (each set ar-
ranged in one plane, and fixed on a frame which could be made to revolve round a vertical axis), would thus present their brightest effect after considerable intervals of darkness; but, by arranging them with their axes slightly inclined, they were made to prolong the light periods and curtail the dark ones. M. Bordier Marcet speaks of this apparatus with all the satisfaction generally felt by inventors; but it is no difficult matter to identify its economic effect with that of the common paraboloidal mirrors. It is obvious, that all the rays which fall from a true focal point on the three reflectors \(\mathrm{AC}, \mathrm{BC}, \mathrm{A}^{\prime} \mathrm{C}^{\prime}\), \(\mathrm{B}^{\prime} \mathrm{C}^{\prime}\), and MN , are merely those which would fall on a single reflector, whose double ordinate and the portion of the abscissa between that ordinate and the focus, are equal to those of the first reflector of the compound system, so that the quantity of light reflected by the three reflectors is neither more nor less than that which would be projected by one. All the difference that can exist is, that in the case of a flame which has a notable size, the surface MN being farther distant than \(a b\), would produce less aberration and, consequently, a very slight increase of intensity in the small portion of the reflected beam of parallel rays due to that part of the compound mirror. We cannot, therefore, sensibly err in rejecting any advantage to be derived from this arrangement as insignificant.*

Spherical mirrors have been employed in lighthouses chiefly when they can be introduced to aid the effect of refracting apparatus; and it will not be necessary to

\footnotetext{
* See Peclet's Traité de l'Eclairage, p. 302, from which fig. 27 is copied.
}
say much of them in this place. I must, however, notice an ingenious proposal of Mr W. H. Barlow,* who suggests placing, in front of the flame, a small spherical reflector, whose centre is coincident with the focus of a paraboloid and whose subtense is the parameter of the generating curve. The small mirror, being somowhat less than a hemisphere, would cause the light falling upon it to be returned through the focus so as to reach the paraboloidal surface, and to be finally reflected from that portion of it which is embraced between the limits of its extreme divergence. If there were no loss of light at the surface of the small mirror, its effect would be to increase the power of the beam of parallel rays by an amount equal to the sum of the rays incident on the spherical surface, but at the same time to diminish it by intercepting a small portion of the light reflected from the paraboloid, equal to a circle whose diameter is the chord of the spherical segment itself. I am not a ware that such a combination has been tried, as it applies most advantageously to reflectors whose span does not exceed the parameter of the generating curve, a form rarely adopted in lighthouses; but it might also be adapted to reflectors which intercept a larger portion of light, by employing a still smaller segment of the sphere. The cone of rays proceeding from the small sphere would also of necessity have great aberration.

Captain Smith, of the Madras Engineers, has described, in the "Professional Papers of the 'Corps of

\footnotetext{
* In an excellent paper, above noticed, on the Illumination of Lighthouses, in the London Transactions, for 1837.
}

Engineers,' * a new system of fixed lights," which consists in placing a flat wick in the focus of one-half of a hollow parabolic spindle generated by the rotation of a parabola about its parameter as a vertical axis. The action of the instrument is obvious, for each vertical section being parabolic, effects a change only in the vertical divergence of the rays incident on it from the focus, and suffers their horizontal direction to remain unaltered; thus each vertical plate of reflected rays passes through the parameter of the curve and illuminates the opposite point of the horizon, by means of a narrow strip or line of light. Two hollow spindles of that form, each lighting \(180^{\circ}\) and facing opposite azimuths, would, therefore, be sufficient to illuminate, in a very feeble manner, the whole horizon of a lighthouse. The author of the paper, however, appears. to contemplate the employment of a series of those mirrors ranged one above another and breaking joint vertically, somewhat in the manner already described in speaking of the arrangement of the paraboloidal mirrors used in fixed lights. The advantages of this mode of illumination are much overrated by Captain Smith, who seems to magnify beyond its real proportion the risk attending the use, in the dioptric apparatus, of a single lamp, whose sudden extinction would deprive at once the whole horizon of the benefit of the light; while, on the contrary, he reckons the security obtained by his arrangement as an advantage of the highest value.
* Vol. v., p. 56.

In certain situations, where no regular establishment of trained light-keepers is maintained, that security may be an object of more importance, and may warrant a greater sacrifice, than is necessary in Great Britain ; but I have no hesitation in saying, that I know of no situation in which the plan proposed by Captain Smith could bear comparison with the mode of illumination for fixed lights by means of the catadioptric instruments of Fresnel.

A brief notice of the manufacture of lighthouse reflectors will conclude the first part of this Treatise. The reflector-plate consists of virgin silver and the purest copper (from the ingot), in the proportion of 6 oz . of silver to 16 oz . of copper. The two metals are in pieces, forming a flat parallelopiped of about nine inches of surface. Being first thoroughly scraped and cleared from rust with a file, they are tied together with wire and placed in the furnace, where they are united by means of a flux composed of burnt borax and nitre, mixed to the consistence of cream. Their thickness is sufficient to admit of their being repeatedly passed through the rolling-mill, so as at last to come out a plate twenty-eight inches square. Every time it is passed through the rollers, the plate is annealed in the furnace before being again pressed. It is then cut into a circular disc ready for working. Great care should be taken to keep the metal perfectly clean during the whole processes of hammering and polishing. The first step towards forming the plate to the curve, is to raise the back or copper side to a slight convexity by beating (with the boxwood mallet, fig. 28 , rounded at
each end, \(c\) and \(d\) ) its inner or silver side upon a large block of beechwood, of a form slightly concave. This beating is begun at the edge of the plate, and gradually reaches the centre. After the disc has been raised to the proper height on the wooden block, the next step is to take it to the horse (fig. 32,

Fig. 28.
 page 125,) where it is beaten with the wooden mallet (fig. 29), its concave face being in contact with the bright steel-head \(a\) (fig. 32), until it has nearly reached the proper height for the reflector, for which the workman has a gauge or mould to guide him; in this course of raising, as it is called, the peened face \(b a b\) (fig. 29) is first used, and then recourse is had to the opposite or flat face for smoothing it after being raised. In this last course of raising, as well as in the process of smoothing the reflector all over, the workman bestrides the horse; and when in this attitude, a boy assists him in manœurring the reflector.

Fig. 29.


Fig. 30.


During the process of raising with the peened side of the mallet, an external mould FGHF (fig. 30), with a needle-point \(P\) at its vertex, to indicate its proper position with reference to the mould, is frequently applied; and allowance is made on the height and diameter of the reflector to meet the expansion of the metal during the hard hammering which is to follow. After each course of the raising with the wooden mallets, the reflector must be annealed in the following manner:-The reflector is first damped with clean water, and its surface dusted over with a powder, composed of one pint of powdered charcoal to one ounce of saltpetre, which is applied by means of a thin flannel bag. The reflector is then put on a clear charcoal fire, where it is turned round as the powder flies off, an effect which indicates that the metal is duly heated. Over-heating is very injurious. When removed from the fire, the reflector is plunged into a large tub, containing what is called the pickle, which is a solution of one quart of vitriol in five or six gallons of water. After this it is washed with clean water, and scoured with Calais sand.

The next step is to put the reflector, thus raised near\(l y\) to its true form, into an iron stool, where a small hole being drilled in its vertex, a circle is described from this point with a beam-compass, so as to cut the paraboloid to the proper size.

The reflector is next hard hammered all over (or planished, as it is technically termed) on the bright steel-head \(a\) (fig. 32), with the planishing hammer (fig. 31); and to facilitate working, the reflector is slung in a flexible frame SS, and counterbalanced by a weight

Fig. 31.


Fig. 33.


Fig. 32.

\(w\), hanging by a cord over the pulleys \(p p\). When the reflector is all planished over, the next process is the smoothing, which is done on the steel-head \(a\), with a lighter hammer (fig. 33), muffled with fine parchment at each end. After it is smoothed comes the finish-
ing, or what is called the filling up to the mould. This is a tedious process; and the workman requires continually to have recourse to the marble table at M , on which he lays the reflector, as shewn in fig. 34, and

Fig. 34.

applies to successive portions of its surface the mould \(g n\), which has a needle-point centred at \(n\), in the small hole drilled in the vertex. During this examination, he marks with a fine slate-pencil those portions of the reflector which do not meet the mould \(g n\). The parts, so marked, are gently gone over with the muffled hammer, until every point touches the mould. This last process requires great caution; for, if any part of the surface be raised above the gauge, it is hardly possible to remedy it. Such a mistake, indeed, can only be corrected by annealing the reflector afresh, and bringing it back to the true form with the mallet; but reflectors so cobbled are never good. The table M (fig. 34) rests on a square box C , in which the tools and moulds are kept.

When thus finished from the hammer, the reflector is put into the apparatus shewn in fig. 35, which is placed at the end of a long dark corridor. RR is a wooden

Fig. 35.

frame fixed to the wall with projecting brackets at K , which support the reflector fixed at E, E, by means of screws, so as always to have a definite position with reference to the bracket B , which carries the lamp and its fountain \(f\), so arranged that its flame may admit of perfect adjustment to the point which ought to be the focus of the reflector. This adjustment is partly effected by the screws S , which serve to raise and depress the level of the burner ; and the lines or marks, \(M, M^{\prime}\) shewn at the sockets J being brought into line, regulate the position of the burner in the horizontal plane of the focus, after it has been raised to the level of that plane by means of the screws at S . The lamp being lighted and thus properly placed, its effect on the reflector's sur-
face is observed by some one stationed at a convenient distance ; and if the whole surface appear luminous the instrument is considered fit for polishing; but if any dark spaces be found in it, the whole reflector must be again carefully tested by means of the mould, and the defective parts remedied in the manner above described.

The next step is to turn over the edge of the reflector, so as to stiffen it. For this purpose it is placed in the matrix \(\mathrm{P}^{p} \mathrm{P}^{p}\) (fig. 36), and the needle-point at V is adjusted by the screw at D , so as just to enter the small hole formerly drilled in the vertex of the reflector. The die-plate PP (worked by means of the arms A A, which turn the screw \(S\) ) then descends and presses the edge over, which is finished with a finely polished tool C, revolving round the axis of the instrument, which, of course, coincides with the centre of the matrix and die. In order to ensure a steady vertical movement of the die-plate P P, cross-arms F F, provided with sockets HH, which slide over the rods G G, G G, are added to prevent any lateral shake or derangement. The whole frame is stiffened by the cross-head in which the screw \(S\) works.

The reflector is then placed on the circular cast-iron table (figs. 37, 38), to which it is attached by the clampscrews S, S. In this position, the bizzle W (fig. 37) and back-belt NAN (fig. 38), are soldered on. After this the reflector is ready for being finally polished; for which purpose, it is placed in a chaise percée, padded round the edges, and is first scoured all over with a piece of pure charcoal of hard wood (generally of pear-tree), and next with a mixture of Florence oil and finely
washed rottenstone, applied by means of a large ball of
Fig. 36.

superfine cloth. It is then carefully cleansed with a piece of fine flannel dipped in Florence oil, and after-

Fig. 37.


Fig. 38.

wards dusted over with the powder of well-washed whiting, and wiped out with a soft cotton cloth. Lastly, it is carefully rubbed by the naked hand, with finely washed rouge and clean water, and wiped with a smooth chamois skin. In all the polishing and cleansing processes, some skill in manipulation is required, as the hand is generally moved in such a manner as to describe successive circles with their planes parallel to the lips of the reflector, and their centres in the axis of the generating curve.

The prices paid to the workmen for the various departments of the reflector-making are generally as fol-lows:-

Raising the plate to the curve, with the wooden
mallet, . . . . . . . £0 10 0
Hammering and smoothing to the mould, . . 150
Finishing in the die, and putting on bizzle and back belt, . . . . . . . 060
Polishing, . . . . . . . 0120
£2 130
The prices paid to the manufacturer were, for the large reflectors of 24 inches aperture, \(£ 43\); for the small ones of 21 inches, \(£ 31,12\) s. The lamp with the slidingcarriage, required for each, costs \(£ 6\).

\section*{A}

\section*{RUDIMENTARY TREATISE}

\author{
ON THE
}

\title{
HISTORY, CONSTRUCTION, and ILLUMINATION
}

OP

\section*{LIGHTHOUSES.}

By
AL.AN STEVENSON, LL.B., F.R.S.E., M.I.C.E., ENGINEER TO TEE BOARD OF NORTHERN LIGRTEOUSEG

> PART II.


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\section*{TREATISE 0N LIGHTHOUSES.}

\section*{DIOPTRIC* SYSTEM OF LIGHTS.}

One of the earliest notices of the application of lenses to lighthouses is in Smeaton's Narrative of the Eddystone Lighthouse, where it is mentioned that a London optician, in 1759, proposed grinding the glass of the lantern to a radius of seven feet six inches. The description is too vague to admit of any conjecture as to the proposed arrangement of the apparatus; but if the lenticular form had been adopted, the fixed light would have been invisible over the greater part of the horizon. About the middle of the last century, however, lenses were actually tried in several lighthouses in the south of England, and in particular at the South Foreland in the year 1752; but their imperfect figure and the quantity of light absorbed by the glass, which was of impure quality and of considerable thickness, rendered their effect so much inferior to that of the paraboloidal reflectors then in use, that after trying some strange combinations of lenses and reflectors, the former were finally aban-

\footnotetext{
* Most probably directly derived from the Greek \(\mathrm{D}_{6}\) orreov, an optical instrument with holes for looking through, whose name is a compound of dock, through, and öorroucu, I see.
}
doned. Lenses were also tried at the lights of Portland, Hill of Howth, and Waterford, by Mr Thomas Rogers, a glass manufacturer in London; who possessed, it is said, the art of blowing mirrors of glass, " and by a new method, silvered over the convex side without quicksilver."*

The object to be attained by the use of lenses in a lighthouse is, of course, identical with that which is answered by employing reflectors; and both instruments effect the same end by different means, collecting the rays which diverge from a point called the focus, and projecting them forward in a beam, whose axis coincides with the produced axis of the instrument. We have already seen that, in the case of reflection, this result is produced by the light being thrown back from a surface so formed as to cause all the rays to proceed in one and the same required direction. In the case of refraction, on the other hand, the rays pass through the refracting medium, and are bent or refracted from their natural course into that which is desired.

The celebrated Buffon, in order to prevent the great absorption of light by the thickness of the material, which would necessarily result from giving to a lens of great dimensions a figure continuously spherical, proposed to grind, out of a solid piece of glass, a lens in steps or concentric zones. This suggestion of Buffon about the construction of large burning glasses, was first executed, with tolerable success, about the year 1780, by the Abbe Rochon; but such are the difficulties attending the pro-

\footnotetext{
* Hutchinson's Practical Seamanship, p. 200. See also the notice of the spherical mirrors made by Messrs Frangois and Letourneau of Paris, in a subsequent part of this volume.
}
cess of working a solid piece of glass into the necessary form, that it is believed the only other instrument ever constructed in this manner, is that which was made for the Commissioners of Northern Lighthouses by Messrs Cookson of Newcastle-upon-Tyne, who imagined that they would thus avoid much risk and uncertainty in uniting the separate zones so as to form a compound lens.

The merit of having first suggested the building of lenses in separate pieces, seems to be due to Condorcet, who, in his Eloge de Buffon, published so far back as 1773, enumerates the advantages to be derived from this method.* Sir David Brewster also described this mode of building lenses in 1811, in the Edinburgh Encyclopedia; and in 1822, the late eminent Fresnel, unacquainted with the suggestions of Condorcet or the description by Sir David Brewster, explained, with many ingenious and interesting details, the same mode of constructing those instruments which he had discovered for himself in 1819. To Fresnel belongs the additional merit of having first followed up his invention, by the construction of a lens, and, in conjunction with MM. Arago and Mathieu, of placing a powerful lamp in its focus, and indeed of finally applying it to the practical purposes of a lighthouse; nor will I omit to record, to his honour, that, in his original Memoire, he frankly

\footnotetext{
* On pourrait même composer de plusieurs pièces ces loupes à échelons; on y gagnerait plus de facilité dans la construction, une grande diminution de dépense, l'avantage de pouvoir leur donner plus d'étendue, et celui d'employer, suivant le bésoin, un nombre de cercles plus ou moins grand, et d'obtenir ainsi d'un même instrument différents dégrès de force.-Eloge dé Buffon, p. 35; CEuvres de Condorcet, tom. iv., Paris 1804.
}
notices the discoveries by which his predecessors had in part anticipated the system of illumination which now justly bears his name.

The great advantages which attend the mode of construction proposed by Condorcet are,-the ease of execution, by which a more perfect figure may be given to each zone and spherical aberration in a great measure corrected, and the power of forming a lens of larger dimensions than could easily be made from a solid piece. Both Buffon and Condorcet, however, chiefly speak of reducing the thickness of the material, and do not seem to have thought of determining the radius and centre of the curvature of the generating arcs of each zone, having contented themselves with simply depressing the spherical surface in separate portions. Fresnel, on the other hand, determined the position of those centres, which continually recede from the vertex of the lens in proportion as the zones to which they refer are removed from its centre; and the surfaces of the zones consequently, are not, as in Buffon's lens, parts of concentric spheres. It deserves notice, that the first lenses constructed for Fresnel by M. Soleil had their zones polygonal, so that the surfaces were not annular, a form which Fresnel considered less accommodated to the ordinary resources of the optician. He also, with his habitual penetration, preferred the planoconvex to the double-convex form, as more easily executed.* After mature consideration, he finally adopted

\footnotetext{
* The plano-convex lens, with its curved side towards the parallel rays, is, moreover, a form producing small spherical aberration, a circumstance which may also have influenced his choice.
}
crown glass, which, notwithstanding its greenish colour, he preferred to flint glass, as being more free from strice. All his calculations were made in reference to an index of refraction of 1.51 , which he had verified by repeated experiments, conducted with that patience and accuracy for which, amidst his higher qualities, he was so remarkably distinguished.* The instruments have received the name of annular lenses, from the figure of the surface of the zones.

To the Dutch belongs the honour of having first followed the French in introducing the system of Fresnel into their lighthouses. The Commissioners of the Northern Lighthouses next followed in the train of improvement; and in 1824, in consequence of a suggestion conveyed to their engineer, Mr Robert Stevenson, in a letter from his friend, General Colby, R.E., they sent him to visit France, and to report upon the lights of that country, which he did, on the 31st December 1824, and by order of the Board also imported lenses from France, for the purpose of experiments. After many trials, he, in a report dated 30th December 1825, recommended the adoption of lenses in the new light of Buchanness, a proposal which, in consideration of certain peculiarities in the distinguishing characteristics of that light, was not acted on. Although many experiments were made with the lenses during the winter of every succeed-

\footnotetext{
* My friend, Mr William Swan, carefully examined, by his new and ingenious method, described in the Edinburgh New Philosophical Journal, January 1844, several specimens of the St Gobain glass (which is now used in the manufacture of the lenses), and found its refractive index to be 1.51793 , the difference between the greatest and least values being only 0.00109 .
}
ing year, it was not until the spring of 1834 that the Commissioners took decisive steps for deciding the question as to the comparative merits of the catoptric and dioptric system, by sending me, in the spring of 1834 , on a mission to Paris, with full power to take such steps for acquiring a perfect knowledge of the dioptric system, and forming an opinion on its merits, as I should find necessary. The singular liberality with which I was received by M. Léonor Fresnel, brother of the late illustrious inventor of the system, and his successor as the Secretary of the Lighthouse Commission of France, afforded me the means of making such a report on my return, as induced the Commissioners to authorise me to remove the reflecting apparatus of the revolving light at Inchkeith, and substitute dioptric instruments in its place. This change was completed, and the light exhibited on the evening of 1st October 1835 ; and so great was the satisfaction which the change produced, that the Commissioners immediately instructed me to make a similar change at the fixed light of the Isle of May, where the new light was exhibited on the 22 d September 1836. The Trinity-House of Deptford Strond followed next in adopting the improved system, and employed me to superintend the construction of a revolving dioptric light of the first order, which was afterwards erected at the Start Point in Devonshire. Other countries, from Gibraltar to the White Sea, and as far east as the Levant, soon began to shew symptoms of interest in this important change; and America, it is believed, is likely soon to adopt active measures for the improvement of her lighthouses. Fresnel, who is al-
ready classed with the greatest of those inventors who extend the boundaries of human knowledge, will thus, at the same time, receive a place amongst those benefactors of the species who have consecrated their genius to the common good of mankind; and, wherever maritime intercourse prevails, the solid advantages which his labours have procured, will be felt and acknowledged.*

A ray of light, in passing obliquely from one transparent body into another of different density, experiences at the point of the intersection of the common surface of the two planes, a sudden change of direction, to which the name of refraction has naturally been given, in connection with the most familiar instance of the phenomenon, which is exhibited by a straight ruler with onehalf plunged into a basin of water, while the other remains in the air. The ruler no longer appears straight, but seems to be bent back or broken at the point where it enters the water. It may not be out of place to call

\footnotetext{
* In justice to General Colby I must notice a statement by Mr Alexander Gordon, in his evidence before the Lighthouse Committee in 1845, to the effect that, " in the year 1833 or 1834 , he was the means of introducing to the Trinity-House and the Northern Lights Commissioners, the lenticulated system of Fresnel." This statement (in so far as concerns the Northern Lights Board, which first adopted the system of Fresnel) is incorrect; and the only communication from that gentleman (which contained a proposal to exhibit a small harbour-light apparatus) was in Sept. 1834, ten years after Mr Robert Stevenson had imported the lenses from Paris, and three months after I had ordered, during my visit to Paris, a catadioptric apparatus of the same kind for the Commissioners of the Northern Lighthouses.
}
attention to the laws which regulate the change of direction in the incident light produced by refraction, which are three in number.
1. Incidence and refraction, in uncrystallised media of homogeneous structure such as glass, always occur in a plane perpendicular to that of the refracting surface.
2. In the same substances, the angle formed with the perpendicular by the ray at its entering the surface of the second medium, has to the angle which it makes with the normal after it has entered the surface, such a relation, that their sines have a fixed ratio, which is called the refractive index. When a ray falls normally on the surface of any substance, it suffers no refraction.
3. The effect of passing from a rare to a dense medium, as from air into water or glass, is to make the angle of refraction less than the angle of incidence; and those angles are measured with reference to a normal to the plane which separates the media at the point of incidence. The converse phenomenon, of course, takes place in the passage from a dense to a rare medium, in which case the angle of incidence is less than the angle of refraction. To this rule there are a few exceptions; for there are certain combustible bodies, such as diamond, whose refractive powers are much greater than other substances of equal density.

The diagram (fig. 39) will serve to render those laws more intelligible. Let a ray of light \(a \mathrm{O}\) meet a surface of water \(n m\) at 0 , it will be immediately bent into the direction \(\mathrm{O} a^{\prime}\); and if, from the centre O , we describe any circle, and draw a line \(b 0 b^{\prime}\), perpendicular to \(n m\); then \(a b\) and \(a^{\prime} b^{\prime}\), perpendiculars drawn to the normal \(b b^{\prime}\), from the points \(a\) and \(a^{\prime}\), where the circle
cuts the incident and refracted rays, will be the sines
Fig. 39.

of the angle of incidence \(b O a\), and of the angle of refraction \(b^{\prime} O a^{\prime}\), and the ratio of those sines to each other, or \(\frac{b a}{b^{\prime} a^{\prime}}\) will be the relative index of refraction for the two media.
4. It may perhaps be added, for convenience, as a fourth law, deducible from the others, that since rays passing from a dense into a rare medium, have their angle of refraction greater than the angle of incidence, there must be some angle of incidence whose corresponding angle of refraction is a right angle ; beyond which no refraction can take place, because there is no angle whose sine can be greater than the radius. In such circumstances, total reflection ensues. For common glass, whose index of refraction is 1.5 , we have (in the case of emergent rays) sine of incidence \(=\frac{\text { sine of refraction }}{1.5}\); but, as no sine can exceed radius or unity, the angle of incidence must be limited to \(41^{\circ} 49^{\circ}\); beyond which total reflection will take place, and the light will re-
turn inwards into the glass, being reflected at its surface.

Thus, if a ray proceed from a point O (fig. 40), within
Fig. 40.

a piece of glass, to a point \(C\), at its surface \(A B\); and if OCb , its incidence, be less than \(41^{\circ} 49\), it will be refracted in some direction \(\mathrm{C} f\); but if this angle be greater than \(41^{\circ} 49\), as \(0 C^{\prime} b^{\prime}\), the ray will be reflected back into the glass in the direction \(\mathrm{C}^{\prime} \mathrm{O}^{\prime}\).

The material hitherto employed in the construction of lighthouse apparatus is crown glass, which, although it possesses a lower refractive power than flint glass, and has, besides, a slightly greenish tinge, offers the great practical advantages of being more easily obtained of homogeneous quality; and, being less subject to deterioration from atmospheric influences, it is peculiarly suitable for use in the exposed situations generally occupied by lighthouses. The refractive index of crown glass, as already noticed, is about \(1 \cdot 5\). There is now, however, some prospect of flint glass, with a low refractive power and considerable purity, being employed.

By protracting the angles of incidence and refraction, in the manner above described, any one may easily satisfy himself of the truth of the following general propositions resulting from those laws:-
1. A ray of light passing through a plate of some diaphanous substance such as glass, with parallel surfaces, suffers no change of direction, but emerges in a line parallel to its original path, merely suffering a displacement, depending on the obliquity of the incident ray, and the refractive power and thickness of the plate. The effect of this displacement is merely to give the ray an apparent point of origin different from the true one. This will be easily understood by the diagram (fig. 41), in which \(a b\) is a normal to the plate,

Fig. 41.

whose surfaces \(x x\) and \(x^{\prime} x^{\prime}\) are parallel, \(r \boldsymbol{r} r\) shews the path of the ray, \(r r\) the displacement, and \(r^{\prime}\) the apparent point of origin resulting from its altered direction.
2. When a ray passes through a triangular prism \(a b c\), the inclination of the faces \(a c\) and \(c b\) causes the
emergent ray \(r^{\prime}\) to be bent towards \(a b\), the base of the
Fig. 42.

prism, in a measure depending on the inclination of the sides of the prism and the obliquity of the incident ray to the first surface.
3. When parallel rays fall on a concave lens, they will, at their emergence, be divergent. The section of

Fig. 43.

the diaphanous body \(a b c d\) may be regarded as composed of innumerable frusta of prisms, having their apices directed towards the centre line \(x r\); and the rays which pass through the centre, being normal to the surface, will be unchanged in their direction, while all the others will (as shewn in the figure), suffer a change of direction, increasing with their distance from the centre, owing to the increasing inclination of the surfaces of the lens as they recede from its axis.
4. Lastly, when divergent rays fall on a convex lens \(a b\), from a point \(f\), called the principal focus, they are

Fig. 44.

made parallel at their emergence; while, conversely, parallel rays which fall on the lens are united in that point.* This effect, which is the opposite of that caused by the concave lens, may be explained in a similar manner, by conceiving the section \(a b\) of the convex lens to be composed of innumerable frusta of prisms arranged with their bases towards the centre of the lens.

Now, it is obvious, that we can derive no assistance, in economising the rays of a lamp for Lighthouse purposes, from concave lenses, whose property is to increase the dispersion of the rays incident on them. With concave lenses, therefore, we have no concern; and we shall confine ourselves to the consideration of the convex or converging lenses.

\footnotetext{
* It is, of course, to be understood that, in the case of a lens whose surfaces are spherical, only the rays incident near its axis can be refracted accurately to a focus.
}

The lens always used in Lighthouses is (for reasons already noticed) plano-convex, and differs from the last only by having a plane and a curve surface, instead of two curve surfaces, whose radii are on opposite sides of the lens. The plano-convex is generally regarded, by writers on optics, as a case of the double convex having one side of an infinite radius. Both forms cause parallel rays to converge to a focus.

We commence with a general view of the relations which subsist between the position of the radiant and the focus.

Let \(\mathrm{Q} q\) be a section of a lens, and \(f \mathrm{~A} r\) its optical axis, or the line in which a ray of light passes unchanged in its direction through the lens, from its being normal to both surfaces, its principal focus \(f\), whether the lens be double convex, as above, or plano-convex, as in fig. 45, is that point where the rays from \(r r r\)
\[
\text { Fig. } 45 .
\]

which fall parallel to the optic axis on the outer face of the lens, meet after refraction at the two faces,-or, to speak more in the language of the art of which we are
treating, the principal focus \(f\) is the point whence the rays of light, proceeding in their naturally divergent course, fall on the inner surface \(Q A q\) of the lens, and are so changed by refraction there and at the outer face, that they finally emerge parallel to the optic axis in the directions \(Q r, q r\). The position of this point depends partly on the refractive power of the substance of which the lens is composed and partly on the curvature of the surface or surfaces which bound it.

It would be quite beyond the scope of this volume to attempt to present the subject of refraction at spherical surfaces before the reader's view in a rigorous or systematic manner, and thus to advance, step by step, to the practical application of refracting instruments, as a means of directing and economising the light in a Pharos. This would involve the repetition, in a less elegant form, of what is to be found in all the works on optics; and instead of this, I am content to refer, where needful, to those works, and shall confine myself simply to what concerns Lighthouse lenses and their use. It would also be superfluous to determine the position of the principal focus of a plano-convex lens, in terms of the refractive index and radius of curvature,* as it
* \(\mathrm{F}=\frac{r}{m-1}\) in which \(r\) is the radius of curvature, and \(m\) is the refractive index.-Coddington's Optics, Chap. VIII. If the radiant be brought near the lens, so as to cast divergent rays on its surface, then the conjugate focus will recede behind the principal focus; and when the luminous body reaches the principal focus in front of the lens, the rays will emerge from its posterior surface in a direction parallel to its axis. If it be brought atill
can be very accurately found in practice by exposing the instrument to the sun, in such a manner that his rays may fall upon it in a direction parallel to its axis. The point of union between the converging and diverging cones of rays (where the spectrum is smallest and brightest), which is the principal focus, is easily determined by moving a screen behind the lens, farther from or nearer to it as may be required. The path of the Lighthouse optician, moreover, generally lies in the opposite direction; and his duty is not so much to find the focal distance of a ready-made lens, as to find the best form of a lens for the various circumstances of a particular Pharos, whose diameter, in some measure, determines the focal distance of the instruments to be employed. All, however, that I shall really have to do is to give an account of what has been effected by the late illustrious Fresnel, who seems to have devoted such minute attention to every detail of the Dioptric apparatus, that he has foreseen and provided for almost every case that occurs in the practice of Lighthouse illumination. His brother, Mons. Léonor Fresnel, who succeeded him in the charge of the Lighthouses of France, with the greatest liberality, put me in possession of the various formulæ used by his lamented predecessor, in determining the elements of those instruments which have so greatly improved the lighthouses of modern days.
nearer the lens, the rays would emerge as a divergent cone. Hence converging lenses can only collect rays into a focus, when they proceed from some point more distant than the principal focus.

Spherical lenses, like spherical mirrors, collect truly into the focus those rays only which are incident near the axis; and it is, therefore, of the greatest importance to employ only a small segment of any sphere as a lens. The experience of this fact, among other considerations, led Condorcet, as already noticed, to suggest the building of lenses in separate pieces. Fresnel, however, was the first who actually constructed a lens on that principle, and fully availed himself of the advantages which it affords; and he has subdivided, with such judgment, the whole surface of the lens into a centre lens and concentric annular bands, and has so carefully determined the elements of curvature for each, that it does not seem likely that any improvement will soon be made in their construction. For the drawings of the great lens, I have to refer to Plate IV., which also contains a tabular view of the elements of its various parts. The central disc of the lens, which is employed in lights of the first order, and whose focal distance is 920 millimètres, or 36.22 inches, is about 11 inches in diameter; and the annular rings which surround it gradually decrease in breadth, as they recede from the axis, from 23 to \(1 \frac{1}{4}\) inches. The breadth of any zone or ring is, within certain limits, a matter of choice, it being desirable, however, that no part of the lens should be much thicker than the rest, as well for the purpose of avoiding inconvenient projections on its surface, as to permit the rays to pass through every part of it with nearly equal loss by absorption. The objects to be attained in the polyzonal or compound lens, are chiefly, as above noticed, to correct the exces-
sive aberration produced by refraction through a hemisphere or great segment, whose edge would make the parallel rays falling on its curve surface converge to a point much ncarer the lens than the principal focus, as determined for rays near the optical axis, and to avoid the increase of material, which would not only add to the weight of the instrument and the expense of its construction, but would greatly diminish by absorption the amount of transmitted light. Various modes of removing similar inconveniencies in telescopic lenses have been devised; and the suggestions of Descartes, as to combinations of hyperbolic and elliptic surfaces with plane and spherical ones, more especially fulfil the whole conditions of the case; but the excessive difficulty which must attend grinding and polishing those surfaces has hitherto deprived us of the advantages which would result from the use of telescopic lenses entirely free from spherical aberration. In lighthouse lenses, where so near an approach to accurate convergence to a single focus is unnecessary, every purpose is answered by the partial correction of aberration which may be obtained, by determining an average radius of curvature for the central disc, and for each successive belt or ring, as it recedes from the axis. In the lenses originally made for Fresnel by Soleil, the zones were united by means of small dowels or joggles of copper, passing from the one zone into the other; but the greater exactness of the workmanship now attained, has rendered it safe to dispense with those fixtures; and the compound lens, as now constructed by Messrs François, Soleil, and Letourneau of Paris, is bound together solely by a metallic
frame and the close union between the concentric faces of the rings, although the surfaces in contact with each other are only \(\frac{1}{4}\) inch in depth (see Plate IV.) It is remarkable, that an instrument, having about 1300 square inches of surface, and weighing 109 lb ., and which is composed of so many parts, should be held together by so slender a bond as two narrow strips of polished glass, united by a thin film of cement.

I will now present to the reader the formulæ employed by Fresnel, to determine the elements of the compound lens,* in the calculation of which two cases occur, viz., the central disc and a concentric ring. The focal distance of the lens and the refractive index of the glass are the principal data from which we start.

Let us begin with the case of the central disc or lens round which the annular rings are arranged. Its principal section is a mixtilinear figure (fig. 46) composed of a segment \(b a c\), resting on a parallelogram \(b c d e\), whose depth \(b d\) or \(c e\) is determined by the strength which is required for the joints

Fig. 46.
 which unite the various portions of the lens. Those particulars have, as already stated, been determined with so much judgment by Fresnel, and the dimensions of the lenses so varied to suit the case of various lights, that nothing in this respect remains to be done by others.

\footnotetext{
* It may be proper to state, that while the formulæ are those of Fresnel, I am responsible for the investigations in the Notes.
}

Referring to fig. 47, we have, for obtaining the radius of the central disc, the following formulæ, in which
\(r=A B\), half the aperture of the lens
\(r^{\prime}=\mathrm{AB}^{\prime}\)
\(\phi=\mathrm{AF}\), the focal distance
\(t^{\prime}=A a\), the thickness of the lens at the vertex
\(t^{\prime \prime}=\mathrm{B} b\), the thickness of the joint
\(\mu=\) the index of refraction
\(\rho=\) the radius of curvature.
\[
\text { Fig. } 47 .
\]


Then for the radius of curvature near the axis we have:
\[
\rho^{\prime}=(\mu-1)\left(\phi+\frac{t^{\prime}}{\mu}\right)
\]
and for that near the margin we have:
\[
\begin{gathered}
\tan i^{\prime}=\frac{r}{\phi} \\
\sin e=\frac{\sin i^{\prime}}{\mu} \\
r^{\prime}=r-t^{\prime \prime} \cdot \tan e \\
\tan i=\frac{r^{\prime}}{\phi} \\
\sin \epsilon=\frac{\sin i}{\mu} \\
\rho^{\prime \prime}=\frac{r}{\mu \sin e} \sqrt{\mu^{2}-2 \mu \cos e+1} \\
\text { and, finally } \rho=\frac{\rho^{\prime}+\rho^{\prime \prime}}{2} *
\end{gathered}
\]
* The following steps lead to the formulæ given in the text. Let APQB (fig. 48) represent a section of the central lens by a

Fig. 48.

plane passing through its axis \(\mathrm{AF} ; \mathrm{F}\) the focus for incident rays; and FQPH the path of a ray refracted finally in the direction PH , parallel to the axis. Let C be the centre of curvature, then PC is a normal to the curve at \(\mathbf{P}\); and, producing PQ to meet the axis in \(G\), we have \(G\) the focus of the rays, after refraction at the surface BQ.

The second case concerns the calculation of the elements of a concentric ring. The section \(a b c d e\) (fig.

Then \(\mu=\frac{\sin \mathrm{PCG}}{\sin \mathrm{GPC}}=\frac{\mathrm{PG}}{\mathrm{CG}}\); and also \(\mu=\frac{\sin \mathrm{QFG}}{\sin \mathrm{QGF}}=\frac{\mathrm{QG}}{\mathrm{QF}}\)
Now, as P approaches A , we have ultimately \(\mathrm{PG}=\mathrm{AG}\),
\[
\mathrm{QG}=\mathrm{BG}, \text { and } \mathrm{QF}=\mathrm{BF} ;
\]

Therefore, putting \(\mathrm{AG}=\boldsymbol{\theta}\) and \(\mathrm{AC}=\rho^{\prime}\)
\[
\mu=\frac{\mathrm{AG}}{\mathrm{CG}}=\frac{\theta}{\theta-\rho^{\prime}} ; \mu=\frac{\mathrm{BG}}{\overline{\mathrm{BF}}}=\frac{\theta-t^{\prime}}{\phi},
\]
from which \(\mu \theta-\mu \rho^{\prime}=\theta\); and \(\mu \phi=\theta-t^{\prime}\) and eliminating \(\theta\), we have \(\mu^{2} \phi+\mu \rho^{\prime}=\mu \phi+t^{\prime}\), whence, as above, \(\rho^{\prime}=(\mu-1)\)
\[
\left(\phi+\frac{t^{\prime}}{\mu} \cdot\right)
\]

But as this value of the radius of curvature, as already stated, is calculated for rays near the axis, it would produce a notable aberration for rays incident on the margin of the lens. In order, therefore, to avoid the effects of aberration as much as possible, a second radius of curvature must be calculated, so that rays incident on the margin of the lens may be refracted in a direction parallel to the axis. This second value of the radius is called \(\rho^{\prime \prime}\) in the text, and is found as follows (referring to fig. 49):

Let \(\mathrm{FB}^{\prime} \boldsymbol{b} \boldsymbol{x}\) be the course of a ray refracted in the direction \(b \boldsymbol{x}\) parallel to the axis \(\mathrm{A} \boldsymbol{x}^{\prime}\). This ray meets the surface AB in the point \(\mathrm{B}^{\prime}\), whose position may be found approximately by tracing the path of the ray FB, on the supposition that the surface of the refracting medium is produced in the directions AB , \(a^{\prime} b^{\prime}\).

Let C be the centre of curvature (See fig. 49.)
\(a=\mathrm{AC} b\) the angle of emergence
\(\eta=\mathrm{B}^{\prime} b \mathrm{C}\) the second angle of refraction
\(\epsilon=\mathrm{B} b \mathrm{~B}^{\prime}\) the first angle of refraction
\(i=B^{\prime}\) FA the first angle of incidence
\(i^{\prime}=\mathrm{BFA}\)
\(e=b^{\prime} \mathrm{B} b\)
50) of one of those rings includes a mixtilinear triangle

Fig. 50.

\(a b e\), and a rectangle \(b c e d\), the thickness \(b c\) being the
\(\mathrm{AB}=r\)
\(\mathrm{AB}^{\prime}=r^{\prime}\)
B \(b=t^{\prime \prime}\) the thickness of the lens at the edge
\(A F=\phi\) the focal distance
Fig. 49.

same as that of the edge of the central disc ; and the elements to be determined are the radius of the curve

Then \(\tan i^{\prime}=\frac{r}{\phi} ; \sin e=\frac{\sin i^{\prime}}{\mu}\)
whence \(b b^{\prime}=t^{\prime \prime} \tan e\) becomes known.
Now, since \(\mathrm{BB}^{\prime}=b b^{\prime}\) nearly, \(\mathrm{AB}^{\prime}=\mathrm{AB}-b b^{\prime}\)
\[
\text { or } r^{\prime}=r-t^{\prime \prime} \tan e .
\]

From this is obtained the angle of incidence \(i\), and the first angle of refraction \(\epsilon\); for \(\tan i=\frac{r^{\prime}}{\phi}\) and \(\sin \epsilon=\frac{\sin i}{\mu}\).
\[
\begin{aligned}
& \text { Next } \mathrm{B}^{\prime} b \mathrm{C}=\mathrm{B} b \mathrm{C}-\mathrm{B} b \mathrm{~B}^{\prime} \text { or } \eta=a-\epsilon \\
& \text { and } \sin a=\mu \sin \eta=\mu \sin (a-\epsilon) \\
& \text { from which, } \sin a \cos \epsilon-\cos a \sin \epsilon=\frac{\sin a}{\mu} \\
& \text { whence } \sin a\left(\cos \epsilon-\frac{1}{\mu}\right)=\cos a \sin \epsilon ; \text { and } \\
& \sin ^{2} a\left(\cos ^{2} \epsilon-\frac{2 \cos \epsilon}{\mu}+\frac{1}{\mu^{2}}\right)=\cos ^{2} a \sin ^{2} \epsilon \\
& =\left(1-\sin ^{2} a\right) \sin ^{2} \epsilon=\sin ^{2} \epsilon-\sin ^{2} a \sin ^{2} \epsilon
\end{aligned}
\]

Then transposing we have
\[
\sin ^{2} a\left\{\left(\cos ^{2} \epsilon+\sin ^{2} \epsilon\right)-\frac{2 \cos \epsilon}{\mu}+\frac{1}{\mu^{2}}\right\}=\sin ^{2} \epsilon
\]
and because \(\left(\cos ^{2} \epsilon+\sin ^{2} \epsilon\right)=1\) we have, by dividing,
\[
\sin ^{2} a=\frac{\sin ^{2} \epsilon}{\left\{1-\frac{2 \cos \epsilon}{\mu}+\frac{1}{\mu^{2}}\right\}}=\frac{\mu^{2} \sin ^{2} \epsilon}{\mu^{2}=2 \mu \cos \epsilon+1}
\]
\[
\text { and } \sin a=\frac{\mu \sin \epsilon}{\sqrt{1-2 \mu \cos \epsilon+\mu^{2}}}
\]

Next, since \(b \mathrm{C}=\frac{a^{\prime} b}{\sin \mathrm{AC} b}=\frac{r}{\sin a}\), putting \(\mathrm{C} b=\rho^{\prime \prime}\),
surface, and the position of the centre of curvature, with reference to the vertex of the lens.

The radius of curvature of the zone may be calculated by the following formulæ, in which (see fig. 51.)

Fig. 51.

and substituting we have
\[
\rho^{\prime \prime}=\frac{r}{\mu \sin \epsilon} \sqrt{\mu^{2}-2 \mu \cos \epsilon+1}
\]
\(r_{1}=A B\) the distance of the outer 费argin of the zone from the axis of the lens
\(r_{2}=\mathrm{AE}\) the distance of the inner margin from the axis
\(l=\mathrm{BE}\) the breadth of the zone \(=r_{1}-r_{2}\)
\(\rho=\) the radius of curvature \(=b \mathrm{C}=m \mathrm{C}\)
\(\phi=\) focal distance AF
\(t=\) thickness of the joint B \(b\)
\(t^{\prime \prime}=\mathrm{B} b\)
\(\mu=\) refractive index of the glass
\(i_{1}=\mathrm{BFA}\)
\(i_{2}=E F A\)
\[
\begin{gathered}
\text { Then } \tan i_{1}^{\prime}=\frac{r_{1}}{\phi} ; \tan i_{2}^{\prime}=\frac{r_{2}}{\phi} \\
\sin e_{1}=\frac{\sin i_{1}^{\prime}}{\mu} ; \sin e_{2}=\frac{\sin i_{2}^{\prime}}{\mu} \\
r_{1}^{\prime}=r_{1}-t^{\prime \prime} \sin e_{1} ; r_{2}^{\prime}=r_{2}-t^{\prime \prime} \sin e_{2} \\
\tan i_{1}=\frac{r_{1}^{\prime}}{\phi} ; \tan i_{2}=\frac{r_{2}^{\prime}}{\phi} \\
\sin \epsilon=\frac{\sin i_{1}}{\mu} ; \sin \epsilon^{\prime}=\frac{\sin i_{2}}{\mu} \\
\sin \alpha=\frac{\mu \sin \epsilon}{\sqrt{\mu^{2}-2 \mu \cos \epsilon+1}} \\
\sin \alpha^{\prime}=\frac{\mu \sin \epsilon^{\prime}}{\sqrt{\mu^{2}-2 \mu \cos \epsilon^{\prime}+1}} ; \eta=\alpha-\epsilon
\end{gathered}
\]
and, taking for the radius of curvature, the mean of \(\rho^{\prime}\) and \(\rho^{\prime \prime}\) the values calculated for the central and marginal rays, we have finally \(\rho=\frac{\rho^{\prime}+\rho^{\prime \prime}}{2}\)
and lastly \(\rho=\frac{2 \cos \epsilon^{\prime}}{2 \cos \left\{\eta+\frac{1}{2}\left(\alpha-a^{\prime}\right)\right\} \sin \frac{1}{2}\left(\alpha-a^{\prime}\right)}\)
which is Fresnel's value of the radius of curvature.*
* The following steps will conduct us to this expression.

Let \(\mathrm{B} b f \mathrm{E}\) (fig. 52) represent the section of a zone by a plane
Fig. 62.


Lastly, the position of C the centre of curvature for a ring is easily determined by two co-ordinates in refer-
passing through the axis of the lens AF, C the centre of curvature, F the radiant point, and \(\mathrm{FB}^{\prime} b x, \mathrm{FE}^{\prime} m x^{\prime}\) the course of the extreme rays which are transmitted through the zone (and the latter of which passes from \(\mathrm{E}^{\prime}\) to \(e\) through a portion of the zone or lens in contact with that under consideration). Then putting
\[
\begin{aligned}
\mathrm{AB} & =r_{1} ; \mathrm{AB}^{\prime}=\eta_{1}^{\prime} ; \mathrm{C} b=\rho \\
\mathrm{AE} & =r_{2} ; \mathrm{AE}^{\prime}=r_{2}^{\prime} ; \mathrm{B} b=t^{\prime} ; \mathrm{BE}=r_{1}-r_{2}=l \\
\epsilon & =\text { the first angle of refraction } b \mathrm{~B}^{\prime} k \\
\eta & =\text { the second angle of refraction } \mathrm{B}^{\prime} b \mathrm{C} \\
\epsilon^{\prime} & =\text { the first angle of refraction } e \mathrm{E} k^{\prime} \\
\eta^{\prime} & =\text { the second angle of refraction } e m \mathrm{C} \\
a & =\text { the angle of emergence } b \mathrm{C} q \\
a^{\prime} & =\text { the angle of emergence } m \mathrm{C} q \\
i_{1}^{\prime} & =\mathrm{BFA}^{2} ; i_{2}^{\prime}=\mathrm{EFA} ; i_{1}=\mathrm{B}^{\prime} \mathrm{FA} ; i_{2}=\mathrm{E}^{\prime} \mathrm{FA} \\
e_{1} & =\mathrm{Bb}_{\mathrm{B}} ; \mathrm{B}^{\prime} ; e_{2}=\mathrm{E} e \mathrm{E}^{\prime} .
\end{aligned}
\]

Proceeding exactly as in the case of the central lens we shall have
\[
\begin{gathered}
\tan {i_{1}^{\prime}}_{1}=\frac{\mathrm{BA}}{\mathrm{AF}}=\frac{r_{1}}{\phi} ; \tan i_{2}^{\prime}=\frac{\mathrm{EA}}{\mathrm{AF}}=\frac{r_{2}}{\phi} \\
\sin e_{1}=\frac{\sin i_{1}^{\prime}}{\mu} ; \sin e_{2}=\frac{\sin i^{\prime}{ }_{2}}{\mu} \\
r_{1}^{\prime}=r_{1}-t^{\prime} \sin e_{1} ; r_{2}^{\prime}=r_{2}-t^{\prime \prime} \sin e_{2} \\
\tan i_{1}=\frac{r_{1}^{\prime}}{\phi} ; \tan i_{2}=\frac{r_{2}^{\prime}}{\phi} \\
\sin \epsilon=\frac{\sin i_{1}}{\mu} ; \sin \epsilon^{\prime}=\frac{\sin i_{2}^{\prime}}{\mu}
\end{gathered}
\]
\(\sin a=\frac{\mu \sin \epsilon}{\sqrt{\mu^{2}-2 \mu \cos \epsilon-1}} ;\) and \(\sin a^{\prime}=\frac{\mu \sin \epsilon^{\prime}}{\sqrt{\mu^{2}-2 \mu \cos \epsilon^{\prime}+1}}\)
Now, the angle \(b \mathbf{C} m=a-a^{\prime}\) from which (since the triangle \(b m \mathrm{C}\) is isosceles) \(b m \mathrm{C}=90^{\circ}-\frac{1}{2}\left(a-a^{\prime}\right)\); also, in the triangle \(b m e\), the angle
ence to their origin, \(A\), which is the vertex of the lens (see last fig., 52 ), by the equations :
\[
\begin{aligned}
& \mathrm{CG}=\rho \cdot \sin \alpha-a b=\rho \cdot \sin \alpha-r_{1} \\
& \mathrm{CQ}=\rho \cdot \cos \alpha-q \mathrm{Q}=\rho \cdot \cos \alpha-t^{\prime \prime}
\end{aligned}
\]

The elements of each successive zone are determined in the same manner. The annular lens of the first order of lights in Fresnel's system consists, as already stated, of a central dise 11 inches in diameter, and ten concentric rings, all of which have a common principal focus, where the rays of the sun meet after passing through the lens. With such accuracy are those rings and the disc ground, and placed relatively to each other, that the position of the actual conjugate focus of the entire surface of the compound lens, differs in a very small degree from that obtained by calculation.

The tests generally applied for examining the lenses
\[
\begin{gathered}
b m e=b m \mathrm{C}-e m \mathrm{C}=90^{\circ}-\frac{1}{2}\left(a-a^{\prime}\right)-\eta \\
\text { and } b e m=k^{\prime} e \mathrm{E}^{\prime}=90^{\circ}-\epsilon^{\prime}
\end{gathered}
\]

We have therefore in the triangle \(b m e\)
\[
b m=\frac{b e \sin b e m}{\sin b m e}=\frac{l \cos \epsilon^{\prime}}{\cos \left\{\eta+\frac{1}{2}\left(a-a^{\prime}\right)\right\}}
\]
and in \(b m \mathrm{C}\)
\[
\begin{gathered}
b \mathrm{C}=\frac{b m \sin b m \mathrm{C}}{\sin b \mathrm{C} m}=\frac{l \cos \epsilon^{\prime} \cos \frac{1}{2}\left(a-a^{\prime}\right)}{\cos \left(\eta+\frac{1}{2}\left(a-a^{\prime}\right) \sin \left(a-a^{\prime}\right)\right.} \\
=\frac{l \cos \epsilon^{\prime} \cos \frac{1}{2}(a-a)}{\cos \left\{\eta+\frac{1}{2}\left(a-a^{\prime}\right)\right\} 2 \sin \frac{1}{2}\left(a-a^{\prime}\right) \cos \frac{1}{2}\left(a-a^{\prime}\right)} \\
\text { from which, putting } b \mathrm{C}=\rho
\end{gathered}
\]
\[
p=\frac{l \cos \epsilon^{\prime}}{2 \cos \left\{\eta+\frac{1}{2}\left(a-a^{\prime}\right)\right\} \sin \frac{1}{2}\left(a-a^{\prime}\right)}
\]
used in lighthouses, is to find the position of the conjugate focus behind the lens, due to a given position of a lamp in front of it. This test depends on the following considerations:-Draw a line from an object 0 in front
\[
\text { Fig. } 53 .
\]

of a lens, to any point \(Q\) in the lens; and from \(A\), the centre of the lens, draw \(A R\) parallel to \(O Q\), and cutting a line RF \(r\) which passes through the principal focus \(F\), at right angles to the axis of the lens; then join the points \(Q\) and \(R\), and produce the line joining them: \(I\), the image of \(O\) must be in that line. In the same way, draw a line from \(O\) to \(q\), another point in the lens on the other side of its axis, and parallel to it draw Ar from the centre of the lens, cutting the plane of the principal focus in \(r\). Join \(q r\), in which line the image will lie; and hence the intersection of OR and \(q r\), in I, will be the point in which the image of O is formed; or will be the conjugate focus of the lens due to the distance OA. This mode will serve to give the distance of the conjugate focus of a lens (neglecting its thickness) for rays falling on its surface at any angle.

We shall suppose QA (fig. 53) to represent the half of a lens, and remembering the conditions described in reference to the last figure, we shall at once perceive the truth of the following analogy (fig. 54):-
\(\mathrm{OA}: \mathrm{AF}: \mathrm{AQ}: \mathrm{FR}: \mathrm{AI}: \mathrm{FI}\), and putting \(\mathrm{OA}=\delta\), \(\mathrm{AI}=\phi^{\prime}\), and \(\mathrm{AF}=\phi\), we have \(\delta: \phi:: \phi^{\prime}: \phi^{\prime}-\phi\), and,

Fig. 54.

consequently, \(\delta \phi^{\prime}-\delta \phi=\phi \phi^{\prime}\); and hence the following equations, which express the relations subsisting between the principal focus of the lens and the distance of any object and its corresponding image :
\(1 s t\), To find the principal focal distance of a lens from the measured position of its object and its image refracted through it, we have, \(\phi=\frac{\delta \phi^{\prime}}{\delta+\phi^{\prime}}\).
\(2 d\), For the distance of the object, when that of the image is known, we have, \(\delta=\frac{\phi \phi^{\prime}}{\phi^{\prime}-\phi^{\prime}}\).
\(3 d\), For the position of the image, when that of the object is known, we have, \(\phi^{\prime}=\frac{\delta \phi}{\delta-\phi}\).

In testing lenses, of course, it is this last equation which we use, because the value of \(\phi\) or the principal focus is always known, and is that whose accuracy we wish to try, while \(\delta\) may be chosen within certain limits at will. I have found that the best mode of proceeding is the following:-In front of the lens \(Q q\) (see fig. 55 ) firmly fixed in a frame, place a lamp at \(O\) at the distance of about 50 yards. Calculate the value of \(\phi^{\prime}\) due to 50 yards, which in this case is equal to \(\mathrm{AF}^{\prime}, \mathrm{OA}\)
being equal to \(\delta\); and move a screen of white paperbackwards and forwards until you receive on it the

Fig. 55.

smallest image that can be formed, which is at the point where the cones of converging and diverging rays meet. The image will always increase in size whether you approach nearer to the lens or recede farther from it, according as you pass from the converging into the diverging cone of rays, or vice versa; and hence the intermediate point is easily found by a very little practice. The distance from the centre of the lens to the face of the screen, which must be adjusted so as to be at right angles to a line joining the centre of the lens and the lamp, is then measured; and its agreement with the calculated length of \(\phi^{\prime}\), is an indication of the accuracy of the workmanship of the lens. When the measured distance is greater than the calculated \(\phi^{\prime}\), we know than the lens is too flat; and it is on this side the error generally falls. On the other hand, when \(\phi^{\prime}\) is greater than the measured distance, we know that the lens has too great convexity. I have only to add, that an error of \(\frac{2}{60}\) on the value of \(\phi^{\prime}\) may be safely admitted in lighthouse lenses; but I have had many instruments made by M. François Soleil, whose error
fell below \(\frac{z^{2}}{8}\) of \(\phi^{\prime}\). Owing probably to the mode of grinding, the surfaces of all the lenses I have yet examined are somewhat too flat.

In applying lenses to the flame of a lighthouse lamp, similar considerations must guide us in making the necessary arrangements, as in the case of reflectors. We have already seen that the size of the flame and its distance from the surface of a mirror have an important practical bearing on the utility of the instrument, and that the divergence of the resultant beam materially affects its fitness for the purpose of a lighthouse. So also, in the case of the lens, unless the diameter of the flame of the lamp has to the focal distance of the instrument a relation such as may cause an appreciable divergence of the rays refracted through it, it could not be usefully applied to a lighthouse; for, without this, the light would be in sight during so short a time, that the seaman would have much difficulty in observing it. To determine the amount of this divergence of the refracted beam, therefore, is a matter of great practical importance, and I shall briefly point out the conditions which regulate its amount, as they are nearly identical with those which determine the divergence of a paraboloìdal mirror illuminated by a lamp in its focus. The divergence, in the case of lenses, may be described as the angle which the flame subtends at the principal focus of the lens, the maximum of which, produced at the vertex of Fresnel's great lens by the lamp of four concentric wicks, is about \(5^{\circ} 9^{\prime}\).

This will be easily seen by examining the annexed figure (56), in which \(Q q\) represents the lens, \(A\) its
centre, F the principal focus, \(b \mathrm{~F}\) and \(b^{\prime} \mathrm{F}\) the radius of
Fig. 56.

the flame ; then is the angle \(b \mathrm{~A} b^{\prime}\) equal to the maximum divergence of the lens. \(\operatorname{Sin} b \mathrm{AF}=\frac{b \mathrm{~F}}{\mathrm{AF}}=\sin b^{\prime} \mathrm{AF}=\) \(\frac{\text { Rad. of flame }}{\text { Focal distance }}\); and twice \(b \mathrm{AF}=\) the whole divergence at A. Then for the divergence at the margin of the lens, or at any other point, we have, \(\mathrm{FQ}=\sqrt{\left(\mathrm{AQ}^{2}+\mathrm{AF}^{2}\right)}\) and \(Q x=\sqrt{\left(\mathrm{QF}^{2}+\mathrm{F} x^{2}\right)}\); and for any angle at Q , we have \(\sin \mathrm{FQ} x=\frac{\mathrm{F} x}{\mathrm{FQ}}\).

On the subject of the illuminating power of the lenses, it seems enough to say, that the same general principle regulates the estimate as in reflectors. Owing to the square form of the lens, however, there is a greater difficulty in finding a mean focal distance whereby to correct our estimate of the angle subtended by the light, so
as to equate the varying distance of the several parts of the surface; but, practically, we shall not greatly err if we consider the quotient of the surface of the lens divided by the surface of the flame as the increased power of illumination by the use of the lens. The illuminating effect of the great lens, as measured at moderate distances, has generally been taken at 3000 Argand flames, the value of the great flame in its focus being about 16, thus giving its increasing power as nearly equal to 180. The more perfect lenses have produced a considerably greater effect.

The application of lenses to lighthouses is so obvious as to require little explanation. They are arranged round a lamp placed in their centre, and on the level of their focal plane in the manner shewn in Plate V.,* and thus form, by their union, a right octagonal hollow prism, circulating round the flame which is fixed in the centre, and shewing to a distant observer successive flashes or blazes of light, whenever one of its faces crosses a line joining his eye and the lamp, in a manner similar to that already noticed in describing the action of the mirrors. The chief difference in the effect consists in the greater intensity and shorter duration of the blaze produced by

\footnotetext{
* Plate V. skews the nature of the mechanical power which gives movement to the lenses. It consists of a clockwork movement driven by a weight which sets in motion a disc bearing brackets that carry the lenses. All this, however, can be seen from that Plate; and I am unwilling to expend time in a detailed explanation of what is obvious by inspection.
}
the lens; which latter quantity is, of course, proportional to the divergence of the resultant beam. Each lens subtends a central horizontal pyramid of light of about \(46^{\circ}\) of inclination, beyond which limits the lenticular action could not be advantageously pushed, owing to the extreme obliquity of the incidence of light; but Fresnel at once conceived the idea of pressing into the service of the mariner, by means of two very simple expedients, the light which would otherwise have uselessly escaped above and below the lenses.

For intercepting the upper portion of the light, he employed eight smaller lenses of 500 mm . focal distance ( 19.68 inches) inclined inwards towards the lamp, which is also their common focus, and thus forming, by their union, a frustum of a hollow octagonal pyramid of \(50^{\circ}\) of inclination. The light falling on those lenses is formed into eight beams rising upwards at an angle of \(50^{\circ}\) inclination. Above them are ranged eight plane mirrors, so inclined (see Plate V.) as to project the beams transmitted by the small lenses into the horizontal direction, and thus finally to increase the effect of the light. In placing those upper lenses, it is generally thought advisable to give their axes an horizontal deviation of \(7^{\circ}\) or \(8^{\circ}\) from that of the great lenses and in the direction contrary to that of the revolution of the frame which carries the lenticular apparatus. By this arrangement, the flashes of the smaller lenses precede those of the large ones, and thus tend to correct the chief practical defect of revolving lenticular lights by prolonging the bright periods. The elements of the subsidiary
lenses depend upon the very same principles, and are calculated by the same formulæ as those given for the great lenses. In fixing the focal distance and inclination of those subsidiary lenses, Fresnel was guided by a consideration of the necessity for keeping them sufficiently high to prevent interference with the free access to the lamp. He also restricted their dimensions within very moderate limits, so as to avoid too great weight. Their focal distance is the same as that for lenses of the third order of lights.

Owing to certain arrangements of the apparatus which are necessary for the efficiency of the lamp, but a small portion of those rays which escape from below the lenses can be rendered available for the purposes of a Lighthouse : and any attempt to subject them to lenticular action, so as to add them to the periodic flashes, would have led to a most inconvenient complication of the apparatus. Fresnel adopted the more natural and simple course of transmitting them to the horizon in the form of flat rings of light, or rather of divergent pencils, directed to various points of the horizon. This he effected by means of small curved mirrors, disposed in tiers, one above another, like the leaves of a Venetian blind-an arrangement which he also adopted (see Plate VI.) for intercepting the light which escapes above as well as below the dioptric belt in fixed lights. Those curved mirrors are, strictly speaking, generated (see fig. 57) by portions such as \(a b\), of parabolas, having their foci coincident with F , the common flame of the system. In practice, however, they are formed as portions of a curved surface, ground
by the radius of a circle, which osculates the given pa-
Fig. 57.

rabolic segment.* The mirrors are plates of glass, sil-
* To find the radius and centre of a circle, which shall osculate a given parabola, whose focus is in F , draw normals to the curve from. \(p\) and P , meeting in O , and draw \(\mathrm{N} e\) parallel to

Fig. 58.

vered on the back and set in flat cases of sheet-brass. They are suspended on a circular frame by means of
a tangent of the curve, or to \(p \mathrm{P}\), then PO or \(p \mathrm{O}\) is the radius required. Now we have similar triangles \(\mathrm{P} p d\) and Nen , and PH and \(p h\) are (proximate) ordinates; hence we have the following analogies:-
\[
\begin{aligned}
& \mathrm{P} d: \mathrm{P} p:: \mathrm{PH}: \mathrm{PN} \\
& \mathrm{~N} e: \mathrm{N} n:: \mathrm{PH}: \mathrm{PN}
\end{aligned}
\]

Hence compounding those ratios (in which \(\mathrm{P} d=\mathrm{N} n\) nearly)
\[
\begin{array}{r}
\mathrm{N} e: \mathrm{P} p_{p}:: \mathrm{PH}^{2}: \mathrm{PN}^{2} \\
\text { also } \mathrm{N} e: \mathrm{P} p^{2}:: \mathrm{NO}: \mathrm{PO},
\end{array}
\]
(for \(\mathrm{OP} p\) and N oe are similar triangles)
\[
\begin{gathered}
\mathrm{PH}^{2}: \mathrm{PN}^{2}:: \mathrm{NO}: \mathrm{OP}, \\
\text { then } \mathrm{PN}^{2}-\mathrm{PH}^{2}=\mathrm{HN}^{2} \\
\text { and } \mathrm{PO}-\mathrm{NO}=\mathrm{NP}, \\
\text { therefore } \mathrm{HN}^{2}: \mathrm{PN}^{2}:: \mathrm{NP}: \mathrm{PO}, \\
\text { and finally, } \mathrm{PO}=\frac{\mathrm{PN}^{3}}{\mathrm{HN}^{2}} .
\end{gathered}
\]

Then put \(\mathrm{FP}=\mathrm{HC}=\mathrm{FN}=\rho ; \mathrm{HN}=\rho-z\);
then as \(\mathrm{FP}^{2}-\mathrm{FH}^{2}=\mathrm{PH}^{2}=\rho^{2}-z^{2}\)
\[
\begin{aligned}
\mathrm{PN}^{2}=\mathrm{PH}^{2}+\mathrm{HN}^{2} & =\left(\rho^{2}-z^{2}\right)+\left(\rho^{2}-2 \rho z+z^{2}\right) \\
& =2 \rho^{2}-2 \rho z \\
\mathrm{PN} & =\sqrt{2 \rho(\rho-z)} \\
\text { Therefore } \mathrm{PO} & =\frac{\sqrt{ }\{2 \rho(\rho-z)\}^{3}}{(\rho-z)^{2}} \\
& =\sqrt{\frac{\{2 \rho(\rho-z)\}^{3}}{(\rho-z)^{4}}} \\
\text { and finally, } \mathrm{PO} & =2 \sqrt{2} \sqrt{\frac{\rho^{3}}{\rho-z}}
\end{aligned}
\]
screws which, being attached to the backs of the brass cases, afford the means of adjnsting them to their true

In order to test the accuracy of the workmanship of the mirrors, recourse must again be had, as in the case of the lenses and parabolic mirrors, to the formula of conjugate foci, in which we shall call \(\mathrm{R}=\) the radius of curvature of the mirror \(\mathrm{M} m\) (fig. 59); \(a=\) the distance of a light, \(f\), which is arbitrarily placed in front of the mirror; and \(b=\) the distance of a moveable screen S , on which the rays reflected from the mirror may converge in a focus. We must find the distance \(b\), at which, with any given distance \(a\), such convergence should take place.

Fig. 59.

\[
\begin{aligned}
& f \mathrm{M}^{\prime}=a \\
& \mathrm{SM}^{\prime}=b \\
& \mathrm{OM}^{\prime}=\mathrm{R}
\end{aligned}
\]

Then (because \(f\) MS is bisected by OM, and for points near the vertex of the mirror at \(\mathrm{M}^{\prime}\) )
\[
\begin{aligned}
& \mathrm{SM}^{\prime}: f \mathrm{M}^{\prime}:: \mathrm{SO}: 0 f \\
& \text { or } b: a:: \mathrm{R}-b: a-\mathrm{R} \\
& a b-\mathrm{R} a=\mathrm{R} b-a b .
\end{aligned}
\]

From which \(b=\frac{\mathrm{R} a}{2 a-\mathrm{R}}\), the distance required, in which an error of \(\frac{1}{3 \delta}\) (of its whole length) may be safely admitted.
inclination, so that they may reflect objects on the horizon of the lighthouse to an observer's eye placed in the common focus of the system.

At such times when the horizon cannot be seen, the mirror may be placed, by means of a clinometer, with a spirit-level set to the proper angle, which may be easily mechanically determined as follows: Draw a line from

Fig. 60.

the focus \(F\) through the point \(O\), where the centre of the mirror is to be, producing it beyond that point to a convenient distance at I ; through O draw HOH , parallel to the horizon FH; bisect IOH by MOM, which coincides with a tangent to the mirror at its centre O ; and MOH is the angle required to be laid off, or its complement.

Having once contemplated the possibility of illuminating Lighthouses by dioptric means, Fresnel quickly perceived the advantage of employing for fixed lights a lamp placed in the centre of a polygonal hoop, consisting of a series of refractors, infinitely small in their length and having their axes in planes parallel to the horizon. Such a continuation of vertical sections, by refracting the rays proceeding from the focus, only in the vertical direction, must distribute a zone of light equally brilliant in every point of the horizon. This effect will be easily understood, by considering the middle vertical section of one of the great annular lenses, already described, abstractly from its relation to the rest of the instrument. It will readily be perceived that this section possesses the property of simply refracting the rays in ondane coincident with the line of the section, and in a direction parallel to the horizon, and cannot collect the rays from either side of the vertical line; and if this section, by its revolution about a vertical axis, becomes the generating line of the enveloping hoop above noticed, such a hoop will of course possess the property of refracting an equally diffused zone of light round the horizon. The difficulty, however, of forming this apparatus appeared so great, that Fresnel determined to substitute for it a vertical polygon, composed of what have been improperly called cylindric lenses, but which in reality are mixtilinear prisms placed horizontally, and distributing the light which they receive from the focus nearly equally over the horizontal sector which they subtend. This polygon has a sufficient number of sides to enable it to give,
at the angle formed by the junction of two of them, a light not very much inferior to what is produced in the centre of one of the sides; and the upper and lower courses of curved mirrors are always so placed as partly to make up for the deficiency of the light at the angles. The effect sought for in a fixed light is thus obtained in a much more perfect manner, than by any conceivable combination of the paraboloidal mirrors.

An ingenious modification of the fixed apparatus is also due to the inventive mind of Fresnel, who conceived the idea of placing one apparatus of this kind in front of another, with the axes of the cylindric pieces crossing each other at right angles. As those cylindric pieces have the property of refracting all the rays which they receive from the focus, in a direction perpendicular to the mixtilinear section which generates them, it is obvious that if two refracting media of this sort be arranged as above described, their joint action will unite the rays which come from their common focus into a beam, whose sectional area is equal to the overlapped surface of the two instruments, and that they will thus produce, although in a disadvantageous manner, the effect of a lens. It was by availing himself of this property of crossed prisms, that Fresnel invented the distinction for lights which he calls a fixed light varied by flashes; in which the flashes are caused by the revolution of cylindric refractors with vertical axes ranged round the outside of the fixed light apparatus already described.
Having been directed by the Commissioners of the Northern Lighthouses to convert the fixed catoptric
light of the Isle of May, into a dioptric light of the first order, I proposed that an attempt should be made to construct the belt for the refracting part of the apparatus of a form truly cylindric instead of polygonal ; and this task was successfully completed by Messrs Cookson of Newcastle in the year 1836. The disadvantage of the polygon lies in the excess of the radius of the circumscribing circle over that of the inscribed circle, which occasions an unequal distribution of light between its angles and the centre of each of its sides; and this fault can only be fully remedied by constructing a cylindric belt, whose generating line is the middle mixtilinear section of an annular lens, revolving about a vertical axis passing through its principal focus. This is, in fact, the only form which can possibly produce an equal diffusion of the incident light over every part of the horizon.

I at first imagined that the whole hoop of refractors might be built between two metallic rings, connecting them to each other solely by the means employed in cementing the pieces of the annular lenses; but a little consideration convinced me that this construction would make it necessary to build the zone at the lighthouse itself, and would thus greatly increase the risk of fracture. I was therefore reluctantly induced to divide the whole cylinder into ten arcs, each of which being set in a metallic frame, might be capable of being moved separately. The chance also of any error in the figure of the instrument has thus a probability of being confined within narrower limits; whilst the rectification of any defective part becomes at the same time more easy. One other
variation from the mode of construction at first contemplated for the Isle of May refractors, was forced upon me by the repeated failures which occurred in attempting to form the middle zone in one piece; and it was at length found necessary to divide this belt by a line passing through the horizontal plane of the focus. Such a division of the central zone, however, was not attended with any appreciable loss of light, as the entire coincidence of the junction of the two pieces with the horizontal plane of the focus, confines the interception of the light to the fine joint at which they are cemented. With the exception of those trifling changes, the idea at first entertained of the construction of the instrument was fully realised, at the very first attempt, in the manufactory of Messrs Cookson. At a subsequent period the central zones were formed in one piece; and I also greatly improved the arrangement of the apparatus, by giving to the metallic frames which contain the prisms, a rhomboidal,* instead of a rectangular form. The junction of the frames being thus inclined from the perpendicular, do not in any azimuth intercept the light throughout the whole height of the refracting belt, but the interception is confined to a small rhomboidal space, whose area is inversely proportional to the sine of the angle of inclination; and when the helical joints are formed between the opposite angles of the rectangular

\footnotetext{
* The form is not exactly rhomboidal, but is a portion of a flat helix intercepted between two planes, cutting the enveloped cylinder at right angles to its axis.
}
frames, the amount of intercepted light becomes absolutely equal in every azimuth.*

Such an apparatus is shewn in Plate VII.; and the accompanying diagram (fig. 61) shews an elevation ABCD , a section BD , and a plan ABD , of a single

Fig. 61.

panel of this improved compound belt. AC and BD are the diagonal joints above described. Time and per-

\footnotetext{
* See my Report on the Refractors of the Isle of May Light, 8th October 1836.
}
severance, and the patience and skill of Monsieur François Soleil, whom I urged to undertake the task, were at length crowned with success; and I had the satisfaction at last of seeing a fixed-light apparatus, of a truly cylindric form, with its central belt in one piece, and the joints of each panel inclined to the horizon at such an angle as to render the light perfectly equal in every azimuth.

The loss of light by reflection at the surface of the most perfect mirrors, and the perishable nature of the material composing their polish, induced me, so far back as 1835, in a Report on the Light of Inchkeith, which had just been altered to the dioptric system, to propose the substitution of totally reflecting prisms, even in lights of the first order or largest dimensions. In this attempt I was much encouraged by the singular liberality of M. Léonor Fresnel, to whose friendship (as I have often, with much pleasure, acknowledged) I owe all that I know of dioptric lighthouses. He not only freely communicated to me the method pursued by his distinguished brother Augustin Fresnel, in determining the forms of the zones of the small apparatus, introduced by him into the Harbour Lights of France, and his own mode of rigorously solving some of the preliminary questions involved in the computacions but put me in possession of various important suggestions, which substantially embrace the whole subject. The Table (at page 66), contains the result of my calculations of the forms of the zones of the first order, which are verifications of those of M. Fresnel ; and the first catadioptric apparatus ever constructed, through the ardour and per-
severance of M. François Soleil, on so magnificent a scale, was fitted up in the Skerryvore Lighthouse. On the 23d December 1843, M. Fresnel announced, in a letter to me, the complete success which had attended a trial of the apparatus at the Royal Observatory of Paris, whereby it appeared that the illuminating effect of the cupola of zones, was to that of the seven upper tiers of mirrors of the first order, as 140 to 87 . Nothing can be more beautiful than an entire apparatus for a fixed light of the first order, such as that shewn in Plate VII. It consists of a central belt of refractors, forming a hollow cylinder 6 feet in diameter, and 30 inches high; below it are six triangular rings of glass, ranged in a cylindrical form, and above a crown of thirteen rings of glass, forming by their union a hollow cage, composed of polished glass, 10 feet high and 6 feet in diameter! I know no work of art more beautiful or more creditable to the boldness, intelligence, and zeal of the artist.

I must now endeavour to trace the various steps by which the elements of the zones given in the Tables (at page 66) have been determined ; and this, I fear, I cannot do without considerable prolixity of detail. Referring to Plates VI., VII., and VIII., in which F shews the flame, RR, the refractors, and MRM and MRM, the spaces through which the light would escape uselessly above and below the lens, but for the corrective action of the mirrors MM, which project the rays falling on them to the horizon, I have to observe that a similar effect is obtained, but in a more perfect manner, by means of the zones ABC and \(\mathrm{A}_{2} \mathrm{~B}_{2} \mathrm{C}_{2}\) (fig. 62, on page 49), whose action on the divergent rays of the lamp
causes the rays \(\mathrm{FC}, \mathrm{FB}\) and \(\mathrm{FC}_{2}, \mathrm{FB}_{2}\) to emerge horizontally, by refracting them at the inner surfaces BC , \(\mathrm{B}_{2} \mathrm{C}_{9}\), reflecting them at \(\mathrm{AB}, \mathrm{A}_{2} \mathrm{~B}_{2}\), and a second time refracting them at \(\mathrm{AC}, \mathrm{A}_{2} \mathrm{C}_{2}\).

The problem proposed is, therefore, the determination of the elements and position of a triangle ABC , which,

Fig. 62.

by its revolution about a vertical axis, passing through the focus of a system of annular lenses or refractors in F, would generate a ring or zone capable of transmitting in an horizontal direction by means of total reflection, the light incident upon its inner side BC from a lamp placed in the point F . The conditions of the question are based upon the well-known laws of total reffection, and require that all the rays coming from the focus \(\mathbf{F}\) shall be so refracted at entering the surface BC , as to meet the side BA at such an angle, that instead of passing out they shall be totally reflected from it, and pass-
ing onwards to the side CA shall, after a second refraction at that surface, finally emerge from the zone in an horizontal direction. For the solution of this problem, we have given the positions of F the focus, of the apex C of the generating triangle of the zone, the length of the side BC or CA, and the refractive index of the glass. The form of the zone must then be such as to fulfil the following conditions:-
1. The extreme ray FB must suffer refraction and reflection at B , and pass to C , where being a second time refracted, it must follow the horizonal direction CH .
2. The other extreme ray FC must be refracted in C and passing to A , must at the point be reflected, and a second time refracted, so as to follow the horizontal course AG (see fig. 63, page 52).

These two propositions involve other two in the form of corollaries.
1. That every intermediate ray proceeding from \(\mathbf{F}\), and falling upon \(B C\) in any point \(E\), between \(B\) and C , must, after refraction at the surface BC in E into the direction EW, be so reflected at W from AB into the direction WI, that being parallel to BC , it shall, after a second refraction in I, at the surface AC, emerge horizontally in the line IK.

And, 2. That the paths of the two extreme rays must therefore trace the position of the generating triangle of the zone.

To these considerations it may be added, that as the angles BCH and FCA are each of them solely due to the refraction at C , as their common cause, they must
be equal to each other, and BCA being common to both the remaining angle \(\mathrm{ACH}=\) the remaining angle BCF.

We naturally begin by the consideration of the lowest ray FC , whose path being traced gives the direction of the two refracting sides BC and AC , leaving only the direction of the reflecting side BA to be determined. I shall not now explain the reason for neglecting entirely the consideration of the reflecting side at present, as I could not do so without anticipating what must be more fully discussed in the sequel; but I may content myself with stating that as the positions of BC and AC depend upon the direction of the incident ray FC, and on the refractive index of the glass, this part of the investigation may be carried on apart from any interference with the reflecting side.

As we know the relation existing between the angles of incidence and refraction, we might determine the relative positions of the sides \(A C\) and \(B C\), by means of successive corrections obtained by protraction, tracing the paths of the rays from the horizontal directions backwards through the zones to the focus. This method, however, depends entirely upon accurate protraction, and is therefore unsatisfactory as a final determination, or if employed for any other purpose than that of affording a rough approximation to the value of the angle, a knowledge of which may occasionally save trouble in the employment of more exact means of determination. A little practice, however, enables one without such aid to make a first guess very near the truth.

Referring to fig. 63, which shews the first and second

Fig. 63.

zones of the upper series, we have
\[
\tan \mathrm{LCF}=\frac{\mathrm{FL}}{\mathrm{CL}} ;
\]
and if we make
the known angle, \(\mathrm{SCF}=\boldsymbol{a}\)
\[
\left.\begin{array}{rl}
\mathrm{OCF}=\xi= & \text { the complement of } \mathrm{BCF}=\text { the } \\
& \text { angle of incidence for } \mathrm{FC}
\end{array}\right) \begin{aligned}
& \mathrm{DCO}=\gamma=\text { angle of refraction } \\
& \mathrm{LCF}=\theta=\left(\mathrm{HCF}-90^{\circ}\right)=\left(2 a-90^{\circ}\right) \\
& \mathrm{SCD}=(\alpha+\gamma-\xi)
\end{aligned}
\]

And \(m=\) the index of refraction for crown glass,
we obtain the means of determining the angles \(\gamma\) and \(\xi\) in two equations, which are based upon the relation between the angles of incidence and refraction, and on
the interdependence of the various angles about \(C\). These primary equations are:
\[
\begin{aligned}
& \sin \xi=m \cdot \sin \gamma \\
& \quad \text { and } \\
& * \gamma=2 \xi-\theta\left(\text { making } 2 \alpha-90^{\circ}=\theta\right)
\end{aligned}
\]

Eliminating \(\gamma\) between these two equations, we obtain :
\[
\sin \xi=m \cdot \sin (2 \xi-\theta)
\]
an expression, which, after various transformations of circular functions, assumes the form
* The truth of the first of these equations \((\sin \xi=m \cdot \sin \gamma)\) which merely expresses the ratio of the sines of the angles of incidence and refraction is obvious; but owing to the great number of small angles about \(C\), a little consideration may be required to enable one to perceive the truth of the second. I therefore subjoin the steps by which I reached it. It is obvious (see fig. 63), that as ACH and BCF are equal, the line SC bisecting HCF must bisect ACB. But the production of AC clearly gives SCD opposite and equal to ACW and SCD is by construction \(=(a-\xi+\gamma)\) \(=(a+\gamma-\xi)\), and, therefore, ACB, which is twice ACW or \(\mathrm{SCD}=(2 a+2 \gamma-2 \xi)\). Now, by construction OC is a normal to the refracting surface CB and its production \(\mathrm{C} g\) gives \(\mathrm{AC} g=\gamma\). But \(\gamma=\mathrm{ACB}-g \mathrm{CB}=(2 a+2 \gamma-2 \xi)-g \mathrm{CB}=(2 a+2 \gamma-2 \xi)\) \(-90^{\circ}\), hence
\[
\gamma=\{2 a+2 \gamma-2 \xi\}-90^{\circ},
\]
and \(\gamma-2 \gamma=-\gamma=-2 \xi+\left(2 a-90^{\circ}\right)\) by transposition, and finally changing signs, we have, as above,
\[
\begin{aligned}
\gamma & =2 \xi-\left(2 a-90^{\circ}\right) \\
& =2 \xi-\theta .
\end{aligned}
\]
\[
\begin{aligned}
& \sin ^{4} \xi-\frac{1}{m} \sin \theta \cdot \sin ^{3} \xi+\left(\frac{1}{4 m^{2}}-1\right) \cdot \sin ^{2} \xi \\
& +\frac{1}{2 m} \sin \theta \cdot \sin \xi+\frac{1}{4} \sin ^{2} \theta=0^{*}
\end{aligned}
\]
* This expression is equivalent to that of M. Fresnel, but owing to a simplification in the fractional coefficients, it is not literally the same. I was led to it by the following steps, starting from the original equation \(\sin \xi=m \sin (2 \xi-\theta)\).
\[
\begin{aligned}
\sin \xi & =m \sin (2 \xi-\theta) \\
& =m\{\sin 2 \xi \cdot \cos \theta-\cos 2 \xi \sin \theta\} \\
& =m \cos \theta \cdot \sin 2 \xi-m \sin \theta \cdot \cos 2 \xi \\
& =m \cos \theta \cdot 2 \sin \xi \cdot \cos \xi-m \sin \theta \cdot\left\{1-2 \sin ^{2} \xi\right\} \\
& =2 m \cos \theta \cdot \sin \xi \cdot \cos \xi-m \sin \theta+2 m \sin \theta \cdot \sin ^{2} \xi
\end{aligned}
\]

Therefore,
\[
m \sin \theta+\sin \xi-2 m \sin \theta \cdot \sin ^{2} \xi=2 m \cos \theta \cdot \sin \xi \cdot \cos \xi
\]

Then:
\[
\begin{aligned}
& m^{2} \sin ^{2} \theta+2 m \sin \theta \cdot \sin \xi-4 m^{2} \sin ^{2} \theta \sin ^{2} \xi \\
& +\sin ^{2} \xi-4 m \sin \theta \cdot \sin ^{3} \xi+4 m^{2} \sin ^{2} \theta \cdot \sin ^{4} \xi \\
& =4 m^{2} \cos ^{2} \theta \cdot \sin ^{2} \xi\left(1-\sin ^{2} \xi\right) \\
& =4 m^{2} \cdot \cos ^{2} \theta \cdot \sin ^{2} \xi-4 m^{2} \cos ^{2} \theta \sin ^{4} \xi .
\end{aligned}
\]

Hence we have
\[
\begin{aligned}
& m^{2} \sin ^{2} \theta+2 m \sin \theta \cdot \sin \xi+\left(1-4 m^{2}\right) . \\
& . \sin ^{2} \xi-4 m \sin \theta \cdot \sin ^{3} \xi+4 m^{2} \sin ^{4} \xi=0
\end{aligned}
\]

Then dividing by \(4 m^{2}\) and arranging according to powers of \(\xi\), we have as above:
\[
\begin{gathered}
\sin ^{4} \xi-\frac{1}{m} \sin \theta \cdot \sin ^{3} \xi+\left(\frac{1}{4 m^{2}}-1\right) \cdot \sin ^{2} \xi \\
+\frac{1}{2 m} \cdot \sin \theta \cdot \sin \xi+\frac{1}{4} \sin ^{2} \theta=0
\end{gathered}
\]

The solution of this equation, which is of the fourth degree, is somewhat tedious; but as the root, which will satisfy the optical conditions of the question, must be the sine of an angle, and necessarily lies between zero and unity; and as the protraction, if conducted with due care in the manner already described, affords the means of at once assuming a probable value of \(\xi\) not very distant from the truth, the labour of the calculation, in this particular case, is not quite so great as might be expected. But notwithstanding all the abridgments of which the particular case admits, considerable trouble is involved, and a corresponding risk of error incurred, in merely introducing the numerical values into the equation preparatory to its solution; and any other method requiring a less amount of arithmetical labour is, of course, to be preferred. In practice I therefore made use of the following method of approximating to the root of the equation, which was suggested to me by a friend.

If the equation \(\sin \xi-m \sin (2 \xi-\theta)=0\) (see page 53) be regarded as an expression for the error, when the true value of \(\xi\) which would satisfy the equation has been introduced into its first member, we may consider any error in the value of \(\xi\) as expressed by the equation:
\[
\sin \xi-m \cdot \sin (2 \xi-\theta)=\boldsymbol{\epsilon}
\]
and differentiating this expression we have:
\[
\begin{aligned}
d \epsilon & =\cos \xi \cdot d \xi-2 m \cos (2 \xi-\theta) \cdot d \xi \\
& =\{\cos \xi-2 m \cos (2 \xi-\theta)\} \cdot d \xi
\end{aligned}
\]

Then dividing by the differential coefficient we obtain
\[
d \xi=\frac{d \epsilon}{\cos \xi-2 m \cos (2 \xi-\theta)}
\]

But when \(\xi\) becomes \(\boldsymbol{\xi}+d \xi, \epsilon\) will also become \(\epsilon+d \boldsymbol{\epsilon}\); but \(\epsilon+d \epsilon=0\)
therefore \(d \epsilon=-\epsilon\)
hence by substitution we have
\[
\begin{aligned}
d \xi & =\frac{-\epsilon}{\cos \xi-2 m \cos (2 \xi-\theta)} \\
& =\frac{-\{\sin \xi-m \sin (2 \xi-\theta)\}}{\cos \xi-2 m \cos (2 \xi-\theta)} \\
d \xi & =\frac{-\sin \xi+m \sin (2 \xi-\theta)}{\cos \xi-2 m \cos (2 \xi-\theta)}
\end{aligned}
\]

By "substituting, therefore, in this last equation the known values of \(m\) and \(\theta\), and the assumed value of \(\xi\), a correction is obtained, which being applied to \(\xi\) and the same process repeated, new corrections may be found until the value of \(d \xi\) falls within the limits of error, which may be considered safe in the particular case. I need hardly say, that where so great a body of flame is employed as in the lights of the first order, these limits are soon passed, more especially as one acquires by a little experience the means of guessing a value of \(\xi\) not very far from the truth. It is this method I have employed in calculating the Tables of the zones (given at p. 66). in which I have on all occasions, though, perhaps, with needless exactness, pushed my angular determinations to seconds.

Having in this manner determined the angle BCF, he obtuse angle BCA of the generating triangle of the
zone is easily and directly deduced by the following expression, which results from the obvious relations existing among the known angles about \(\mathbf{C}\); and we have (see fig. 63, p. 52).
\[
\mathrm{BCA}=90^{\circ}+\gamma=90^{\circ}+2 \xi-\theta .
\]

We next proceed to consider the form of BA, the reflecting side of the zone, which is a point of the greatest consequence, as an error in the inclination of any part of its surface is doubled in the resulting direction of the reflected rays. The conditions of the question require, that every ray EW, after reflection at the surface \(A B\), shall, like WI, be parallel to the first ray, which is reflected in the direction BC , and after a second refraction at C, emerges horizontally in CH . But, let us trace backwards the rays as they emerge in their horizontal directions IK, and it is obvious that if BA be made a straight line, then will every ray EW meet the first refracting side BC at the same angle, and there suffering the same refraction, they will go on parallel to each other, and never meet in the focus \(F\). This convergence to \(F\), which is a necessary condition of the problem, may, however, be produced by a curvature of AB, such that all the rays shall have a degree of convergence before falling on BC , sufficient to cause them to be finally refracted, so as to meet in F. On this account, they will occupy less space in passing through BC , than they did in passing through AC ; and thus BC will be shorter than AC by some quantity which shall give to that part of AB which is at B the amount of downward inclination required for causing the ray BF finally to converge to \(F\); and the line joining \(B\) and \(A\) must be a
curve, every point of which has its tangent inclined so as to serve the same purpose.

To trace tangents to this curve, is therefore the next step in the process. The direction of the first tangent AZ depends upon very simple considerations; and all that is necessary to be done is to draw a line AU (fig. 64), parallel to BC (which is the parallel to the direc-

Fig. 64.

tion of the reflected rays), and forming an angle CAU, which is, of course, equal to the inclination of the extreme ray refracted by CB at C , with the rays reflected from the arc which we have yet to trace. The line AX bisecting this angle, must therefore be a normal to the reflecting surface at A , and AB drawn perpendicular to AX , is consequently a tangent to the reflecting arc.

We must next find the direction of the second tangent \(\mathrm{Z} b\), which must be so inclined that the ray \(\mathrm{F} b\) will, after refraction at \(b\), be reflected into the direction \(b \mathrm{C}\); but as the rigorous determination of this is difficult, I shall simply describe two approximations suggested to me by M. Leonor Fresnel. The first method is based upon assuming the inclination of the ray refracted at \(b\) to the ray refracted at C as equal to:
\(\frac{b \mathrm{FC}}{m}\)
(in which expression, \(m\) is the refractive index of the glass); a supposition which obviously differs very little from the truth, as small ares may be assumed as nearly equal to their sines. Now, it will be recollected, that the rays refracted at \(C\) and \(b\), must be reflected at \(A\) and \(b\), in a direction parallel to \(\mathrm{C} b\), and therefore the inclination of the reflecting surfaces, or that which should be formed by the tangents ZA and \(\mathrm{Z} b\), being half that of the incident rays, is, according to the assumption, equal to \(\frac{b \mathrm{FC}}{2 m}\), which may be expressed by \(\frac{s}{3} b \mathrm{FC}, m\) being equal to \(1 \cdot 51\). But as the inclination of the two radii \(A X\) and \(B X\) is equal to the inclination of the tangents of the reflecting surfaces to which they are normals, we obtain for the excess \(\mathrm{B} \beta\) of the secant of the reflecting arc over its radius the following expression:
\[
\mathrm{B} \beta=\frac{1}{2} \mathrm{AB} \cdot \tan \frac{1}{3} \mathrm{BFC} \cdot *
\]

\footnotetext{
* The following steps will shew the mode of obtaining this expression : Suppose (fig 65, on following page) F \(n\) to be a ray in-
}

The value of \(B\) or \(B b\) gives, of course, the direction of the second tangent \(\mathrm{Z} b\) (which must be equal in length to
cident on the surface \(B C\) very near \(b\) or \(B\) (which, although exaggerated in the figure for more easy reference, are close together), and let this ray \(\mathrm{F} n\) be refracted in the direction \(n \mathrm{O}\), and draw \(n n^{\prime}\) parallel to CA, the ray which is refracted at \(C\), then will \(n^{\prime} n \mathrm{O}=m . b \mathrm{FC}=\frac{9}{3} b \mathrm{FC}\). But the tangent AZ should make with the tangent \(b \mathrm{Z}\) an angle equal \(\frac{1}{3} b \mathrm{FC}\), or one-half the inclination of the rays refracted at \(b\) and \(\mathbf{C}\), which are afterwards, by the agency of those tangents, to be reflected in the directions parallel to \(b \mathrm{C}\) and to each other. Hence we have AX \(b\) (which is the inclination of the normals to those tangents),
\[
\text { or } \mathrm{AX} b=\mathrm{BZ} b=\frac{b \mathrm{FC}}{2 m}=\frac{b \mathrm{FC}}{3} \text { nearly. }
\]

But putting AXB (fig. 64, p. 58) for \(\mathrm{AX} b\), and BFC for \(b \mathrm{FC}\)
Fig. 65.


AZ ), whence we easily deduce the chord of the reflecting side A \(b\).

The second mode proposed by M. Fresnel, and that which I found most convenient in practice, consists in forming successive hypotheses as to the length of the side BC , and tracing the path of the incident ray FB , which being refracted at B , so as to make with the normal BK (fig. 66, on the following page), an angle \(=\mathrm{KBY}=y^{\prime}\), and finally reflected in the direction BC, must make the angle \(\mathrm{YBZ}=\mathrm{MBC}\). In the same figure MBZ is a tangent to the reflecting surface at B , and KBF is the angle of
(a supposition which may be safely made when the differences are so small), and founding upon the analogy
\[
\begin{aligned}
& \mathrm{AX}: \mathrm{AB}:: \mathrm{R}: \tan \mathrm{AXB}, \text { we have } \mathrm{BA}=\mathrm{AX} \cdot \tan \mathrm{AXB} \\
&=\mathrm{AX} \cdot \tan \frac{1}{3} \mathrm{BFC} . \text { Then } \\
& \mathrm{AB}^{2}=\mathrm{B} \beta(\mathrm{~B} \beta+2 \mathrm{AX})=\mathrm{B} \beta^{2}+2 \mathrm{~B} \beta \cdot \mathrm{AX}
\end{aligned}
\]
and neglecting \(\mathrm{B} \beta^{2}\), which is very small, we have:
\[
\begin{gathered}
\mathrm{BA}^{2}=\mathrm{B} \beta .2 \mathrm{AX} \text { nearly, } \\
\text { hence } \mathrm{B} \beta=\frac{\mathrm{BA}^{2}}{2 \mathrm{AX}} \\
\text { But as above } \mathrm{AX}=\frac{\mathrm{BA}}{\tan \frac{1}{3} \overline{\mathrm{BFC}}}
\end{gathered}
\]
and substituting this value of \(A X\) we obtain :
\[
\mathrm{B} \beta=\frac{\mathrm{BA}^{2}}{2 \frac{\mathrm{BA}}{\tan \frac{1}{3} \mathrm{BFC}}}
\]
hence we have, as in the text,
\[
\mathrm{B} B=\frac{1}{2} \mathrm{BA} \cdot \tan \frac{1}{3} \mathrm{BFC}
\]
incidence of the ray BF before its refraction at B . If
Fig. 66.

\(\mathrm{KBF}=x^{\prime}\), and the angle of incidence of \(\mathrm{FC}=\mathrm{ECF}=x\), we have BFC (which is the inclination of those rays to each other, and must be equal to the difference of their angles of incidence to the same surface) \(=x-x^{\prime}\), whence knowing \(x\), we easily find a value of \(x^{\prime}\) corresponding to the length of BC . Then for finding the angle of refraction \(\mathrm{KBY}=y^{\prime}\) we have :
\[
\sin y^{\prime}=\frac{\sin x^{\prime}}{m}
\]

Now, if FB be refracted, so as to make with the reflecting side an angle equal to ZBY, it must (if the position of B be rightly chosen) be reflected so as to follow BC , thus making \(\mathrm{MBC}=\mathrm{YBZ}\), and calling each of
these angles \(=\mu\), we have the right angle NBZ made up of \(\mu+y^{\prime}+\) NBK. But NBK clearly equals \(\mu\), because it is the inclination of the normals to BC and BZ , and hence \(y^{\prime}+2 \mu=90^{\circ}\). This, therefore, forms a crucial test for the length of BC. I may only remark, that we already know the numerical value of \(y\); and that of \(\mu\) is easily found, for \(\mu=\mathrm{CBA}+\mathrm{ABM}=\mathrm{CBA}+\mathrm{BAM}=\) \(\mathrm{CBA}+(\mathrm{MAC}-\mathrm{BAC})=\mathrm{CBA}+\frac{1}{2}\left(180^{\circ}-v\right)-\mathrm{BAC}\). Thus knowing \(\mu\) and \(y^{\prime}\), we have only to see whether
\[
\left(y^{\prime}-2 \mu\right)-90^{\circ}=0
\]

We have now to find the length of the radius \(A X\) or \(b \mathrm{X}\) (see fig. 64, p. 58), which will describe the reflecting surface or arc AZ \(b\), and to determine the position of its centre X . We already know the values of \(y^{\prime}\) and \(y\), the angles of refraction of C and \(b\), and their difference \(y-y^{\prime}\) gives us the inclination of the rays which are to be reflected (into directions parallel to \(\mathrm{C} b\) ) at \(b\) and at \(A\). This quantity is, of course, double the inclination of tangents to the reflecting surface \(A Z\) and \(b \mathrm{Z}\), and of their normals AX and \(b \mathbf{X}\). Again, we have the chord line
\[
\mathrm{Ab}=\frac{\mathrm{AC} \cdot \sin \mathrm{AC} b}{\sin b \mathrm{CA}} ;
\]
and, as above,
\[
\operatorname{AX} b=\frac{1}{2}\left(y-y^{\prime}\right)=\phi
\]

And
\[
\mathrm{AX}=b \mathrm{X}=\rho=\frac{\mathrm{A} b \cdot \sin \frac{1}{2}\left(180^{\circ}-\phi\right)}{\sin \phi}=\frac{1}{2} \mathrm{~A} b \cdot \operatorname{cosec} \frac{1}{2} \phi .
\]

And, lastly, for the co-ordinates to \(\mathbf{X}\), the centre of curvature for the reflecting are, we have
\[
\begin{aligned}
\mathrm{OX} & =\rho \cdot \sin \mathrm{OAX} \\
\text { and } \mathrm{OA} & =\rho \cdot \cos \cdot \mathrm{OAX} .^{*}
\end{aligned}
\]

The positions of the apices \(A\) and \(B\) of the angles of the zones are also easily found in reference to the focus, and are given in the Tables (page 66). In fig. 67

Fig. 67.

we may, in reference to the known position of C , find that of A or B , by simply adding the quantities AH , HC , and BK, to \(\mathrm{C} y\) or \(\mathrm{C} x\), and by deducting CK from \(\mathrm{C} y\); while it is obvious that those quantities are respectively proportional to the length of the known sides AC and BC , modified by the inclination of those sides
* The angle OAX is easily found, as will be seen by referring to fig. 64, p. 58 ; for, AH being horizontal by construction and \(A O\) vertical, \(\mathrm{HAO}=90^{\circ}\); and HAC and CAU being both known, we have
\[
\mathrm{OAX}=90^{\circ}-(\mathrm{HAU}+\mathrm{UAX})=90^{\circ}-\left(\mathrm{HAU}+\frac{1}{2} \mathrm{CAU}\right)
\]
with the horizon. Hence we have \(\mathrm{AH}=\mathrm{AC} \cdot \sin \mathrm{ACH}\); \(\mathrm{HC}=\mathrm{AC} \cdot \cos \mathrm{ACH} ; \mathrm{BK}=\mathrm{BC} \cdot \sin \mathrm{BCK}\); and \(\mathrm{CK}=\) BC. \(\cos\) BCK.

In the process of grinding the zones, it is found convenient for the workman to give a curved form to the refracting sides BC and AC , the one being made convex and the other concave, so that both being ground to the same radius, the convergence of the rays produced by the first shall be neutralized by the divergence caused by the second. By this arrangement we have three points given in space from which, with given radii, to describe a curvilinear triangle whose revolution round the vertical axis of the system generates the zone required. Co-ordinates to those two centres of curvature for the surfaces AC and BC were determined in reference to. the arris A of each zone, and will be found in the Tables (page 66). The mode of finding those co-ordinates is, of course, similar to that already given ; and, the radii being assumed at 4000 millimètres, the co-ordinates are respectively proportional to the sine and cosine of the inclination of the radius at A to the vertical line, which inclination depends upon the relations of known angles around A and C .

The section ABC (fig. 68, p. 67) of the first zone being thus determined, we proceed by fixing the point \(\mathrm{C}_{2}\) of the second zone, which is at the intersection of the horizon \(\mathrm{GAC}_{2}\) with the ray \(\mathrm{FBC}_{2}\) passing through B . This arrangement prevents any loss of light between the adjacent zones. The calculation of the elements of the second and of every following zone, is precisely similar to that for the first.

\section*{TABLES of the Elements of Catadioptric Zones suited for Lights of the First Order in the System of Augustin Fresnel.}

Table I.


Fig. 68.


Table II.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{\[
\begin{gathered}
\text { No. of } \\
\text { Zone. }
\end{gathered}
\]} & \multicolumn{6}{|l|}{Co-ordinates of the Apices of the Generating Triangle, in Millimètres, having the Axis of the System cut by the horizontal plane of the focus of the Annular Lenses, for its origin. (Fig. 69.)} \\
\hline & \(\mathrm{A}_{y}\) & \(\lambda_{*}\) & \(\mathrm{B}_{\mathrm{y}}\) & \(\mathrm{B}_{3}\) & \(\mathrm{C}_{y}\) & C. \\
\hline 1 & 593.369 & 986-260 & \(551 \cdot 481\) & \(831 \cdot 497\) & 525.000 & 920.060 \\
\hline 2 & 663.063 & 957-410 & 617.423 & 808.832 & 593.369 & \(895 \cdot 816\) \\
\hline 3 & \(734 \cdot 313\) & 926.908 & 684.958 & 783.619 & \(663 \cdot 063\) & \(869 \cdot 605\) \\
\hline 4 & 807.357 & 894.222 & \(754 \cdot 267\) & 755.451 & 734.313 & \(840 \cdot 920\) \\
\hline 5 & \(882 \cdot 445\) & 858.858 & 825.547 & 723.921 & 807.357 & 809.337 \\
\hline 6 & \(959 \cdot 847\) & \(820 \cdot 325\) & 899.011 & \(688 \cdot 608\) & \(882 \cdot 445\) & \(774 \cdot 426\) \\
\hline 7 & \(1039 \cdot 852\) & \(778 \cdot 108\) & 974.899 & 649.051 & 959.847 & 735730 \\
\hline 8 & 1122.784 & 731.645 & 1053•471 & 604.730 & \(1039 \cdot 852\) & \(692 \cdot 742\) \\
\hline 9 & \(1209 \cdot 000\) & \(680 \cdot 322\) & \(1135 \cdot 025\) & 555.059 & 1122.784 & 644.900 \\
\hline 10 & 1298.900 & 623.433 & \(1219 \cdot 892\) & \(499 \cdot 354\) & 1209.000 & \(591 \cdot 556\) \\
\hline 11 & 1390.290 & 559.378 & \(1308 \cdot 183\) & \(439 \cdot 428\) & 1288.900 & 531.963 \\
\hline 12 & 1482.755 & 490.169 & \(1398 \cdot 007\) & 374.538 & \(1390 \cdot 290\) & \(467 \cdot 217\) \\
\hline 13 & 1576.048 & 415.991 & 1488.972 & 304.611 & 1482.755 & 397.403 \\
\hline 1 & 593.815 & 985.212 & 550.915 & 831.725 & 525.000 & 920.000 \\
\hline 2 & \(682 \cdot 731\) & 981-810 & 634.862 & \(831 \cdot 183\) & \(610 \cdot 872\) & 920.000 \\
\hline 3 & 779.464 & 978.917 & 726.831 & \(830 \cdot 694\) & 704.730 & 920.000 \\
\hline 4 & 885.046 & 974.441 & 827.926 & \(830 \cdot 261\) & 807.657 & 920.000 \\
\hline 5 & \(1000 \cdot 663\) & \(970 \cdot 608\) & \(939 \cdot 385\) & 829.882 & 920.871 & \(920 \cdot 000\) \\
\hline 6 & 1127.665 & 966.771 & 1062.591 & 829.557 & 1045740 & 920.000 \\
\hline
\end{tabular}

Fig. 69.


Table III.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow{2}{*}{\[
\begin{aligned}
& \text { No. of } \\
& \text { Zone. }
\end{aligned}
\]} & \multirow[t]{2}{*}{} & \multirow[t]{2}{*}{\begin{tabular}{|c|}
\hline bC, \\
Length of \\
Refrnet \\
Reting \\
Side of \\
ZZne in \\
Min \\
mêli. \\
neres. \\
(Fig. 70.)
\end{tabular}} & \multirow[t]{2}{*}{} & \multicolumn{3}{|l|}{Inclination of the Sides of the Generating Triangle to the Vertical Axis of the System. (Fig. 71.)} \\
\hline & & & & Inclination
of AB. & Inelination
of \(\operatorname{BC}\). & \begin{tabular}{l}
C. \\
C.
\end{tabular} \\
\hline 1 & \(160 \cdot 3\) & 92.379 & \(95 \cdot 209\) & 745119 & \(73 \quad 2031\) & \\
\hline 2 & 15 & \(90 \cdot 249\) & 93.0 & 725527 & 743232 & 41 \\
\hline 3 & 15 & 88.729 & \(91 \cdot 434\) & \(70 \quad 5939\) & \(75 \quad 4251\) & 8 \\
\hline & 14 & 87 & \(90 \cdot 424\) & \(69 \quad 0353\) & 765131 & 36 \\
\hline 5 & 14 & 87 & 89.947 & 670812 & 775841 & \(\begin{array}{llll}33 & 24 & 19\end{array}\) \\
\hline 6 & 145.087 & \(87 \cdot 403\) & 89.986 & 651236 & 790427 & 3040 \\
\hline 7 & 14 & 87.977 & 90.536 & 631703 & 800856 & \(27 \quad 5434\) \\
\hline 8 & 144.609 & 89.060 & 91.604 & 612135 & 811214 & \(\begin{array}{lll}25 & 07 & 52\end{array}\) \\
\hline 9 & & 90.671 & 93.209 & 592608 & 821429 & 2220 \\
\hline 10 & 147.089 & \(92 \cdot 843\) & 95 & \(57 \quad 3046\) & \(\begin{array}{llll}83 & 15 & 47\end{array}\) & 931 \\
\hline 11 & \(145 \cdot 361\) & 93.000 & & \(55 \quad 36 \quad 30\) & \(84 \quad 1616\) & 41 \\
\hline 12 & 14 & 93.000 & 95 & 53 & 851424 & 56 \\
\hline 13 & 14 & \(93 \cdot 000\) & 95-127 & \(\begin{array}{lllll}51 & 58 & 53\end{array}\) & 861002 & 1116 \\
\hline 1 & & & 94.806 & \(74 \times 2{ }^{\circ}\) & \(\begin{array}{llll}73 & 38 & 20\end{array}\) & \({ }_{4}^{\circ} 3{ }^{\prime} 78\) \\
\hline 2 & 158.05 & 92.00 & 94 & 722210 & 745306 & 4042 \\
\hline 3 & 156.592 & 92.000 & 94.719 & 702145 & 760559 & \(37 \quad 5432\) \\
\hline 4 & 155.083 & \(92 \cdot 000\) & 94.619 & 682316 & 771621 & 3507 \\
\hline 5 & 153.489 & \(92 \cdot 000\) & 94.488 & 662811 & 782326 & \(32 \quad 2306\) \\
\hline 6 & 151.863 & 92.000 & 94.334 & \(6437 \quad 37\) & 792645 & 294319 \\
\hline
\end{tabular}

Fig. 70.


Fig. 71.


Table IV.
\begin{tabular}{|c|c|c|c|c|c|}
\hline No. of & \[
\begin{aligned}
& \text { Radins of } \\
& \text { inventore } \\
& \text { int } \\
& \text { intilit. } \\
& \text { oere } X, \\
& \text { or } X b \text {. }
\end{aligned}
\] & Horizontsl distance of centre of cmrvatare \(X\) from the axis of
the System in Millimètres
\(=\mathbf{O X}\). & Vertical distance of oentre of curvature X
below the outer arris of the Zone at \(A\) in Mili
metres \(=0 \mathrm{~A}\). & Inclination of the Two \(A\) and \(b\). & \[
\begin{aligned}
& \text { Inclination of } \\
& \text { the Outer } \\
& \text { Radius in A to } \\
& \text { the Vertex. }
\end{aligned}
\] \\
\hline 1 & \(8750 \cdot 19\) & \(3194 \cdot 76\) & 8466.88 & \(1{ }^{\circ} 02{ }^{\circ} 518\) & 143811 \\
\hline 2 & 8252.90 & 3306.80 & \(7909 \cdot 94\) & 10445 & 163211 \\
\hline 3 & \(7850 \cdot 30\) & 3411.75 & 7446.67 & 10622 & \(18 \quad 2711\) \\
\hline 4 & 7527.08 & 3514.22 & 7056.39 & 10751 & 202211 \\
\hline 5 & 7275.23 & 3617.56 & \(6730 \cdot 66\) & 10912 & 221711 \\
\hline 6 & 7082.17 & 3723.81 & \(6459 \cdot 63\) & 11025 & 241211 \\
\hline 7 & 6944.09 & \(3835 \cdot 23\) & \(6234 \cdot 93\) & 111131 & 260711 \\
\hline 8 & 6857.03 & 3954.67 & 6052.36 & 11230 & 280211 \\
\hline 9 & 6817.87 & 4081.70 & \(5902 \cdot 53\) & 11320 & 295711 \\
\hline 10 & 6824.46 & 4226.67 & 5795.68 & 11405 & 315211 \\
\hline 11 & \(6880 \cdot 52\) & \(4385 \cdot 63\) & \(5718 \cdot 52\) & 11237 & \(33 \quad 4711\) \\
\hline 12 & 6980.89 & 4558.83 & \(5672 \cdot 64\) & 11036 & \(\begin{array}{llll}35 & 38 & 59\end{array}\) \\
\hline 13 & 7123.05 & 4747.24 & 5654.92 & 10815 & 372659 \\
\hline 1 & 8674.74 & 3243.48 & 8375.64 & \(1{ }_{1} 0309\) & \(1505 \quad 22\) \\
\hline 2 & 8419.60 & 3456.44 & 8047.72 & 1.0431 & \(17 \quad 05 \quad 32\) \\
\hline 3 & 8278.82 & \(3685 \cdot 41\) & 7823.68 & 10501 & \(\begin{array}{lll}19 & 05 & 13\end{array}\) \\
\hline 4 & \(8252 \cdot 95\) & \(3941 \cdot 89\) & 7701.00 & 10436 & 210424 \\
\hline 5 & \(8335 \cdot 86\) & 4228.06 & 7673.05 & \(1 \begin{array}{ll}1 & 0318\end{array}\) & \(\begin{array}{llll}23 & 00 & 10\end{array}\) \\
\hline 6 & 8522.64 & \(4549 \cdot 97\) & 7732.80 & 10115 & 245143 \\
\hline
\end{tabular}

Fig. 72.


Table V.
\begin{tabular}{|c|c|c|c|c|c|}
\hline & \multicolumn{5}{|c|}{AC, Onter Refracting Surfaces (concavo).} \\
\hline  &  &  &  & \[
\begin{aligned}
& \text { Inelination } \\
& \text { of thatindii } \\
& \text { in A and } .
\end{aligned}
\] & \[
\begin{aligned}
& \text { Inclination of } \\
& \text { Rntidioutater } \\
& \text { the vertex. }
\end{aligned}
\] \\
\hline 1 & 4000.00 & \(3825 \cdot 31\) & 2817.77 & 1 \(21 \begin{array}{lll} \\ 1\end{array}\) & \({ }_{4}^{\circ} 512\)\begin{tabular}{ll}
12 \\
\hline 16
\end{tabular} \\
\hline 2 & ... & \(3923 \cdot 65\) & 2683.55 & 11956 & 475152 \\
\hline 3 & ... & \(4015 \cdot 06\) & 2542.21 & 11834 & 503214 \\
\hline 4 & ... & 4098.54 & 2394.23 & 11742 & 531400 \\
\hline 5 & ... & 4173.08 & 2239.64 & 11718 & 555702 \\
\hline 6 & \(\ldots\) & 423770 & 2078.82 & 11720 & 584115 \\
\hline 7 & \(\ldots\) & \(4291 \cdot 45\) & 1912.18 & 11748 & 612632 \\
\hline 8 & ... & \(4333 \cdot 30\) & \(1740 \cdot 11\) & 11844 & 641246 \\
\hline 9 & ... & \(4362 \cdot 26\) & \(1563 \cdot 10\) & 12006 & 665950 \\
\hline 10 & ... & \(4377 \cdot 24\) & 1381.63 & 12158 & 694736 \\
\hline 11 & ... & 4376.72 & \(1194 \cdot 94\) & 12200 & 723706 \\
\hline 12 & \(\ldots\) & \(4360 \% 0\) & \(1009 \cdot 41\) & 12112 & 752258 \\
\hline 13 & \(\ldots\) & 4329.31 & 828.19 & 12146 & 780302 \\
\hline 1 & \(4000 \cdot 00\) & 3855.83 & \(2785 \cdot 60\) & \(\mathrm{i}^{\circ} 2128\) & \({ }^{\circ} 55^{\circ} 5140\) \\
\hline 2 & ... & 3983.21 & 2644-16 & 12128 & 483715 \\
\hline 3 & \(\ldots\) & \(4104 \cdot 84\) & 2494.82 & 12124 & 512446 \\
\hline 4 & \(\ldots\) & \(4218 \cdot 56\) & 2340.02 & 12120 & 541148 \\
\hline 5 & ... & \(4322 \cdot 95\) & \(2182 \cdot 17\) & 12112 & 565618 \\
\hline 6 & ... & \(4416 \cdot 91\) & 2023.99 & 12104 & 593609 \\
\hline
\end{tabular}

Table VI.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & \multicolumn{5}{|c|}{\({ }_{\text {oc, In }}\) Iner Refracting Sarfaces (convex).} & \[
\Delta,
\] \\
\hline  & \[
\begin{gathered}
\text { Radins of } \\
\text { Cnrvatinre } \\
\text { in Milli- } \\
\text { mêtres. }
\end{gathered}
\] & Horizontal dis tance of centre from the axise the System in Millimètres. & Vertical distance of centre of curvature
below the onter arris of the Zon at \(A\) in Millimètres. & Inclination of the Radii
in C and \(b\). in \(C\) and \(b\). & Inclination of the Onter the Vertex. &  \\
\hline 1 & 4000.00 & 2021•19 & \(3777 \cdot 07\) & \(1{ }_{1} 19 \times 4\) & \begin{tabular}{llll}
15 & 5 & 4 \\
\hline 17
\end{tabular} & 1054.34 \\
\hline 2 & ... & 1918.37 & \(3797 \cdot 40\) & 11734 & 144841 & 1069.02 \\
\hline 3 & ... & 1813.60 & \(3815 \cdot 77\) & 11616 & \(\begin{array}{lll}13 & 39 & 01\end{array}\) & 1087.52 \\
\hline 4 & ... & 1707.55 & 3831.95 & 11526 & 123046 & 1109.85 \\
\hline 5 & ... & 1599•72 & 3846.04 & 11504 & 112347 & 1136.14 \\
\hline 6 & \(\cdots\) & 1489.62 & \(3858 \cdot 14\) & 11508 & \(10 \quad 17 \quad 59\) & 1166.57 \\
\hline 7 & ... & 1376.71 & 3868.30 & 11536 & 91316 & 1201.46 \\
\hline 8 & ... & 1260.38 & 3876.59 & 11632 & 80930 & \(1241 \cdot 16\) \\
\hline 9 & ... & 1139.94 & 3883.03 & 11756 & 70633 & 1286.15 \\
\hline 10 & ... & 1014.67 & \(3887 \cdot 66\) & 11948 & 60419 & 1336.99 \\
\hline 11 & ... & 884.95 & 3893.00 & 11956 & 50346 & 1394.36 \\
\hline 12 & ... & 752.78 & 3897-32 & 11956 & 40538 & 1457.22 \\
\hline 13 & ... & \(618 \cdot 37\) & \(3901 \cdot 10\) & 11956 & 31000 & \(1525 \cdot 43\) \\
\hline 1 & 4000.00 & 2002.52 & \(3781 \cdot 91\) & \(1{ }^{\circ} 101904\) & \({ }^{\circ} 5484208\) & \\
\hline 2 & ... & 1918.55 & 3801.51 & 11904 & 142722 & \\
\hline 3 & ... & 1836.22 & 3818.93 & 11904 & 131429 & \\
\hline 4 & ... & 1756.33 & \(3834 \cdot 20\) & 11904 & 120407 & \\
\hline 5 & ... & 1679 -85 & \(3847 \cdot 38\) & 11904 & 105702 & \\
\hline 6 & & \(1607 \cdot 39\) & 3858.56 & 11904 & 95343 & \\
\hline
\end{tabular}

Note.-The Six co-ordinates of the Generating Triangle of each Zone in the foregoing Tables have the Focus of the Principal Lenses of the System for their origin; it being considered more convenient, in executing the necessary protractions, preparatory to the construction of the grinding apparatus, at once to refer the whole of the grinding machinery to the axis of the apparatus. To prevent the appearance of any inconsistency, however, it is proper to mention, that the radiant points of the series of Zones, do not exactly coincide with the Focus of the Lenses nor with each other; and that to avoid the parallax which the distance of the radiant points from the origin of the co-ordinates would occasion, it is necessary to make some corrections upon the linear dimensions, so as to find the line corresponding to the angles \(\theta, \xi\), and the distance \(\Delta\). In the Hyperpyral series, which stands above the flame, the Zones have the radiant point 10 millimètres above the Focus of the Lenses, and each \(y\) of this series, therefore, requires to be reduced by that amount; while the \(x\) remains unchanged. In the Hypopyral series, which stands below the flame, the Focus or radiant points of each Zone, varies its place in the flame, moving upwards as the Zone is lower, so that the line joining the Zones and the Foci revolves as a radius vector round a point between them. In this way, the \(x^{\prime} s\) remain unaltered; but the \(y^{\prime}\) 's will be lengthened successively by the addition of \(10,14,19,25,32\), and 40 millimètres. As these Tables contain the dimensions of Zones which are intended as an addition to the apparatus of Fresnel, I have adopted the metric scale, so as to render them at once applicable to the existing protractions of that system. It is only necessary to add, that the conversion of millimètres into imperial inches is easily effectd by adding the \(\log\). millimètres to the \(\log . \overline{2} \cdot 59516\), the sum being the \(\log\). of the equivalent of the first term in imperial inches.

The process of grinding the zones, which \(I\) shall here notice, is that which is followed by M. Theodore Letourneau, who now manufactures the apparatus for the Northern Lights Board, in the room of M. François Soleil, who is engaged at St Petersburg in the same work. For this account, the reader is indebted chiefly to notes which were furnished to me by M. Letourneau himself.

The glass used in all the parts of the optical apparatus of the Dioptric Lighthouses is that of St Gobain, whose index of refraction, as already noticed, is \(1 \cdot 51\).
As well on account of the difficulty experienced in producing at all times regular castings of glass from the moulds, as in order to compensate for the frequent accidents, which occur in the first application of the rubbers to the inequalities of the surface of the glass, the castings, whether for rings of lenses or prisms, are made from moulds, exceeding the intended size of the finished pieces by one-eighth part.

We shall take as an example, which is well calculated to illustrate the difficulties of the grinding process, one of the prismatic rings of a Catadioptric Light of the first order. The first operation will be, to take off the rough arris at the angles of the pieces as they come from the moulds, and to reduce to equality the length of each of the four quadrantal prisms or segments by removing from each the quantity that may be necessary to make those four pieces, when placed on a circle, exactly equal to that of the finished zones. Each of them must have an excess of material at the various surfaces just sufficient to insure the rubber having scope enough to remove all the flaws or defects of
the two surfaces to be first ground, which are the (concave and convex) refracting faces of the zone (the sides AC and BC in fig. 68, p. 67).

The pieces must be placed end to end on the horizontal plane or table of the lathe at AA (fig. 73), and

Fig. 73.

must rest on the exterior arris \(\mathbf{A}\) of the reflecting side, on which arris there is ground a narrow plane, whose width is proportionate to the projection of the outer edge beyond the inner edge of the zone, foreshortened by the bevel or inclination of the reflecting side, when resting (as in fig. 73) on the circular iron belt, which is screwed to the table of the lathe provided for its reception. This narrow plane at the arris A should be sufficient to give the prism a solid and regular bearing on the circular iron belt. In this figure (fig. 73) \(a b\) is the vertical axis of the lathe, \(n\) the point from which the horizontal co-ordinate for O , the grinding centre for the
exterior concave refracting surface AC is measured, and \(e \mathrm{AC}\) the direction of the arc swept by the grinding surface. Conversely, \(n^{\prime}\) is the origin of the horizontal coordinate for the grinding centre 0 of the interior convex refracting surface BC , and \(e \mathrm{CB}\) the direction of the arc swept by its grinding surface.* Some skill is required in fixing the prism on the belt, for, on the one hand, there is an obstacle to correct workmanship from the dragging motion of the platform, and on the other, by the unequal subdivision of the weight of the glass, which should be nearly balanced. The narrow plane, already noticed, being perfectly adjusted for all the segments so as to bed them quite level, the circular iron belt on which the ring should be ground, is placed on its platform, in the manner represented in the figure. It should be as truly levelled as possible, otherwise all the subsequent operations will be deranged by it. This iron belt is heated by means of heating pans; and the degree of heat may be practically judged of by the ebullition of drops of water let fall on it. The segments of glass are also at the same time placed in a stove heated with steam, and are generally raised to about \(120^{\circ}\) centigrade. The difference between the time required for the two operations of heating the iron belt and the glass segments is employed in laying or bedding a quantity of cement on the reflecting side of the segment, so as to fill up the angular space between the glass and the iron belt, and also to serve as a seat for the segment in the manner shewn in fig. 73. This operation

\footnotetext{
* The vertical co-ordinates are measured in every case from the level of A the outer arris of the zone.
}
is performed on a plane surface, in order that the lower part of the mastic may be precisely on a level with the narrow plane already ground on the outer arris of the reflecting side. After being sure that the heat is equally spread over every part of the circular iron belt, the segment is arranged on it; and the workman must, at this juncture, exert all his skill in placing the parts of the segment in a position nearly concentric with the belt, or in a truly circular form, making due allowance, however, for the inequalities existing at various parts of the rough material, and at the same time taking care that there should be an interval of at least two millimètres (or about \({ }_{1}^{2}\) th inch) between the ends of each of the two adjacent segments. Without this interval the heat evolved during the polishing would either dilate the glass so much as to cause the ends of the segments to fly into splinters, or make it needful to remove the zone before this should take place, the inevitable consequence of which would be the fracture of the pieces. Those intervals between the segments are filled with statuary's plaster, which must be carefully washed and brushed at each change of the emery employed in grinding.
The exterior diameter of the circular iron belt must be precisely equal to that of the ring, because, if larger, the free movement of the rubbers to and fro on the concave refracting surface AC (fig. 73) could not take place.

By what is already said, it will be obvious that the grinding process is begun at the refracting sides AC and BC , and a few words will shew that this could hardly be otherwise. If a commencement be made on the reflecting side, which appears at first sight more
natural, the consequence is obvious. Having provided for an excess of material in every direction, the segment must consequently be larger than it will be when finished; and the surfaces therefore cannot be true and perfect, except they be ground throughout their entire segmental section, from their centres of curvature, in reference to some given apex of the generating triangle. Now, if the reflecting side were finished first, it might continue to possess this excess of size after being finished, and would, therefore, afford no accurate starting point for the grinding of the other surfaces; it would also present no surface or narrow plane for resting firmly on the iron belt, but would then depend merely on its own finished plane, which, being curved

Fig. 74.

v:
and considerably inclined, would not give a solid bearing for the glass. The other mode of commencing with the two refracting sides, on the contrary, gives a solid bearing on the narrow plane already ground on the reflecting side at A ; and after those surfaces have been ground, and the segments inverted (as shewn in fig. 74), the outer edge of this narrow plane at the arris \(A\), which has been fully defined by the intersection of the finished surface \(A C\) just ground, and also the apex at C, which has been determined by the intersection of AC and BC , combine to fix an accurate starting point for the rubber, in grinding the reflecting surface AB.

Dressing off the rough part of the ring is the first step. This is generally done by means of fixed rubbers, the adjustment of which is more easily regulated than that of the moveable beam or radius of the arc, which is used to give the exact curvature of the surface. Those fixed rubbers are 150 millimètres (nearly 6 inches) wide, by 200 millimètres (nearly 8 inches) long, and are of cast-iron. Three such rubbers are placed at equidistant points of the circle. Two cutters of sheetiron attached to arms placed vertically (as are also those which carry the rubbers), and moving in grooves radiating towards the centre of the lathe, so as to admit of adjustment to suit the varying radii of the zones, serve gradually to abrade the outer and inner arrisses of the segments, so as to prevent the splintering to which, from becoming too sharp, those arrisses, without this precaution, would be liable. Those rubbers are, besides, fixed by stems to frames, in the form of quadrants
of the circle, which allow of a change in the direction of the planes, as occasion may require.

Instead of the siliceous sand formerly used, the powder of pounded freestone is employed, as it is found to wear the tools less, and toform a better preparation for the subsequent grinding operations. It is easy to conceive that the action of the fixed rubbers necessarily produces ruts or inequalities in the circular direction. The operation of rough dressing, therefore, is not finished until, when those first rubbers are removed, the surfaces of the segments have been subject, for the required time, to the action of moveable rubbers, attached to arms working as radii of curvature, in a plane at right angles to the horizontal movement of the lathe, which carries the zones.

The emery grinding is the second step. The form of the segment should be nearly perfect, after the rough grinding is finished. The lathe and the zone are then subjected to an extremely careful washing. Every place where the stone-powder might adhere is dusted. The radius of curvature is verified afresh, agreeably to the co-ordinates in Tables IV., V., and VI.; and emery is used instead of powdered stone; beginning with that called No. 1, which is drawn after suspension in water for one minute. Brushes are used for spreading the emery on the surface of the glass. The quantity ought always to be sufficient to prevent the direct contact of the cast-iron rubber with the glass. Splintering or scratching, which cannot be easily effaced, may result from the neglect of this precaution.

Practice alone, and an eye duly trained by continual experience, can determine the point of time at which each kind of emery must be discontinued. The celerity
of the work depends on circumstances very difficult to appreciate, such as the amount of the pressure of the rubbers, or the degree of accuracy with which the radius of curvature has been adjusted, during the rough grinding. Each kind of emery in succession thus corrects the form of the zone and refines the grain of the surface of the glass; and each change to a fresh material requires the same attention to cleanliness, so as to remove every trace of the substance last used in grinding, and thus to give each successive process its full and legitimate effect. The douci is the fifth and last kind of emery which precedes the polish. It is drawn off after ten minutes' suspension in water, and is extremely fine Before applying it, the greatest care is necessary to in sure cleanliness, as a single grain of any of the preceding kinds of emery might cause scratches, which the polish cannot remove.

Next follows the polish. The same considerations which induce the workman most carefully to cleanse the lathe and everything connected with it, before employing the last emery called the douci, are still more urgent in the case of the final polish. The only change which is made at this last stage of the work is to replace the first rubber by a new one, both longer and wider by about 50 millimètres (nearly two inches). On its lower face is laid, with cement, a piece of soft carpet, whose cdges are fixed to the rubber by means of tat bands of iron, attached with screws. This security, addei to that given by the cement, is necessary to fit it to resist the great pressure it must sustain. A practical question, which experience alone can resolve, occurs at this stage, as to the proper time of commencing
the operation of polishing, which, in the hands of unskilful persons, may be so inopportunely begun as to make that work almost endless. Thus, the mere circumstance of spreading at the beginning too thick layers of rouge, or using unsuitable kinds of carpet, would cut scratches in the glass, and thus perhaps make it necessary to return to the use of the emery called douci. Sometimes, also, if the carpets be not washed at the very time of using them, scratches are formed by the dust which they may contain. This shews, that the use of rouge should be rather sparing than otherwise, at the commencement of the polish; and that the carpet-cloths should be brushed and washed, as M. Letourneau says " twice rather than once." In all cases, the quality of the carpet forms an important element in the success of the working.

When the polish is finished, the ring is detached from the circular belt, simply by the tap of a hammer, on the inner edge of the circle. The division of the zones (which are quarters of the circle) into eighths, is done by means of a sawing machine consisting of a flat cop-per-wheel, one-half millimètre ( \(\gamma_{0}\) inch) in thickness, attached to an arm with a counterpoise. This wheel descends and cuts the zone by means of emery, which is continually applied to it; the direction of the cut is radial. The two halves of the zone are detached from each other, as soon as their weight exceeds the resistance of the part which remains to be sawn.

Lastly comes the adjustment of the prisms in the frames, an operation which is not without risk. Much care is required in handling the sharp arrisses of the
glass, which are very acute and delicate and at the same time lie in a curved direction, which makes them liable to be splintered in the hands of unskilful persons.

The plane vertical surfaces of the annular lenses, and the central band and rings of the dioptric belts of fixed lights, are ground by means of vertical rubbers with a reciprocating movement; and all the plane surfaces are executed by hand on a flat table.

The cement for fixing the glass on the lathe is composed of 8 parts Swedish pitch, and 1 part of wood ashes. The whole is heated in an iron pot until fully liquified and thoroughly mixed. The cement is used almost in a state of ebullition, so that it cannot be handled without the precaution of continually dipping the hands in cold water.

The cement used for the adjustment of the pieces of glass which touch each other consist of 12 parts white lead, 1 part minium or red lead, and 5 parts boiled lintseed oil. The whole is pounded on an iron table by means of a flat mullet, like that used by painters (fig. 75), whose grinding surface is \(a b\), and \(c\) the knob for the hand. This cement is applied liquid so as to offer no resistance to the close union of the pieces,

Fig. 75.
 which it is intended to unite.

The cement for filling up voids, and fixing the rings in the frames, is composed of 12 parts white lead, 3 parts whiting, 1 part minium, and 4 parts boiled lintseed oil. This last composition differs from the former only in the introduction of whiting, which, while like minium it has a
desiccative property, gives more body to the cement and prevents the formation of cracks. The oil is also decreased in quantity, as the cement must be used in a more compact state. The trituration of this cement is performed by means of a cylindric iron roller \(a b\), with a centre-spoke \(c d\) for the hand (fig 76).

Fig. 76.


It is essential for the production of good cement, that the mixture of the ingredients be complete.

The expense of the various parts of the Dioptric apparatus is as follows: Great lens of first order, \(£ 58\) ( 8 of which are required); pyramidal lens and mirror, \(£ 14,12\) s. ( 8 of which are required); catadioptric cupola, for \(360^{\circ}\) of horizon, \(£ 480\); catadioptric rings below lenses, \(£ 360\); panel of dioptric belt for fixed lights of first order, \(£ 56\) (of which 8 are required for the whole circle); apparatus of fourth order, for a fixed light, for whole horizon, \(£ 128\); apparatus of sixth order, for whole horizon, £44. The expense of the mechanical lamp of the first order with four wicks (with framed tripod and adjusting screws, as made for the Scotch lighthouses), is \(£ 30\).

I next proceed to consider what mode should be followed in testing the accuracy of the zones. For this purpose, various expedients suggested themselves, such
as the application of gauges in the form of a radius, having at one end a plate with a triangular space cut through it, equal and similar to the cross section of the zone. The horizontal motion of this arm would, of course, detect the inaccuracies of the successive sections of the inclosed zone. The application of such a gauge, however, seemed difficult, and in order to test the form of the zones, I satisfied myself with using callipers (similar to the sliding rules used by shoemakers) for measuring the length of the sides of the zone, and a goniometer for the angles, which is shewn in the figure (fig. 77), in which ABC represents the prism, with one

Fig. 77.

angle inclosed between the arms AC and AB , moveable round a centre 0 , and \(R R\) the graduated limb. This instrument, however, is inconvenient and defective, as the convexity of the sides AB and BC of the zone requires some skill in getting the arms to be tangents to them.

A practical test, moreover, yet remained to be made of the zon \(3 s\) when fixed in the brass frames, and assembled around the common focus of the system (as shewn in

Plate VIII.), by measuring the final inaccuracy in the path of the rays emergent from them. I have successfully used the following mode. Having mounted the frames containing the zones on a carriage revolving round a small flame placed truly in the common focus, I carefully marked with a piece of soap the centre of the emergent surface AC of each zone (fig. 68, p. 67); and having attached to a vertical rod of metal a telescope, provided with a spirit-level and cross-hairs (for cutting the centre of the image of the flame reflected through the zone) in such a manner as to be capable of sliding on the rod, I observed the cutting of the centre of the flame by the cross-hairs. In the case of any aberration from a normal emergence of the central ray, I had thus the means of at once determining its amount and direction. The telescope was moved up or down, and its horizontal inclination was varied until the axis of the instrument coincided with the direction of the ray emergent from the centre of each zone, which was made to circulate round the flame, the observer noting any change in the position of the reflected image of the flame, and causing an attendant to mark the zones in which the change occurred, that they might again be subjected to separate examination of the same kind, by adjusting the telescope to the error of each. The inclination of the telescope, and the consequent aberration of the ray, was then measured by a graduated arc, with an adjusting spiritlevel, moved by a rack and pinion, which were attached to the telescope. By this method I have succeeded in detecting the inaccurate position of some of the zones in the frame; and the error has been reduced by care-
fully resetting them, so as to diminish considerably the aberration of a great proportion of the emergent rays. Another test, however, remained to be applied to the zones, when finally arranged in their places on the frame; and the mode which I found most convenient in practice was simply to measure the complements of the vertical inclination (given in Table III., page 70) of each surface of the zone, and more especially that of the reflecting surface, by means of the instruments

Fig. 78.

shewn in figures 78 and 79, after the zones were fixed in their place. The figure (No. 78) shews the mode of measuring the inclination of the reflecting side AB of

Fig. 79.

a zone of the upper series; and the second (No. 79) shews the position of the instrument when applied to the reflecting side \(A B\) of a zone of the under series. In those figures, L is a spirit-level ; R , a graduated limb for reading the angular deviation from the true inclination of the tangents to each surface; and SS are studs which rest on the convex surfaces AB and BC of the zones, so as to make the ruler parallel to the tangents of those sides. I have only to add, that I have restricted the error in the position of the reflecting side of the zones to \(50^{\prime}\) as an extreme limit; and I have invariably endeavoured, in altering the position of the zone in the frame, to throw any error on the side of safety, by causing the rays to dip below the horizon, rather than to rise above it.*

\footnotetext{
* In connection with the use of the clinometer, I determined the inclinations of the tangents or chords of the three curve sur-
}

Fig. 80.


The mode of framing the greater zones is nearly the same as that used for the Small Harbour Light apparatus of the fourth order. The chief difference consists in the diagonal framing, which I adopted for supporting the cupola of 13 zones, which, from its great weight, could not be safely made to rest on the dioptric belt below. It is shewn in Plates VII. and VIII., and accords with the mode of jointing the refractors already described. This system has now been rendered still more complete by the adoption of lanterns composed of diagonal framework, afterwards described and shewn at Plate X.

We have next to consider the great Lamp, to the proper distribution of whose light, the whole of the apparatus, above described, is applied. Fresnel immediately perceived the necessity of combining with the dioptric instruments which he had invented, a burner capable of producing a large volume of flame; and the rapidity with which he matured his notions on this subject and at once produced an instrument admirably adapted for the end he had in view, affords one of the many proofs of that happy union of practical with theoretical talent, for which he was so distinguished. Fresnel himself has modestly attributed much of the merit of the invention of this lamp to M. Arago; but that
faces \(\mathrm{AB}, \mathrm{BC}\), and AC of each zone to NP, the axis of the system, by means of the obrious relations of the known angles about C, A, and B. Those inclinations (fig. 80) are shewn by the angles BNO, BON, and CPF; and are given in the Table of the Zones, No. III.
gentleman, with great candour, gives the whole credit to his deceased friend, in a notice regarding lighthouses, which appeared in the Annuaire du Bureau des Longitudes of 1831. The lamp has four concentric burners, which are defended from the action of the excessive heat produced by their united flames, by means of a superabundant supply of oil, which is thrown up from a cistern below by a clockwork movement and constantly overflows the wicks, as in the mechanical lamp of Carcel. A very tall chimney is found to be necessary, in order to supply fresh currents of air to each wick with sufficient rapidity to support the combustion. The carbonisation of the wicks, however, is by no means so rapid as might be expected; and it is even found that after they have suffered a good deal, the flame is not sensibly diminished, as the great heat evolved from the mass of flame, promotes the rising of the oil in the cotton. I have seen the large lamp at the Tour de Corduan burn for seven hours without being snuffed or even having the wicks raised; and, in the Scotch Lighthouses, it often, with Colza oil, maintains, untouched, a full flame for no less a period than seventeen hours.

The annexed diagrams will give a perfect idea of the nature of the concentric burner. The first (fig. 81) shews a plan of a burner of four concentric wicks. The intervals which separate the wicks from each other and allow the currents of air to pass, diminish a little in width as they recede from the centre. The next (fig. 82) shews a section of this burner. \(\mathrm{C}, \mathrm{C}^{\prime}, \mathrm{C}^{\prime \prime}, \mathrm{C}^{\prime \prime \prime}\) are the rack-handles for raising or depressing each wick ; AB
is the horizontal duct which leads the oil to the four

Fig. 81.


Fig. 82.

wicks ; L, L, L, are small plates of tin by which the burners are soldered to each other, and which are so placed as not to hinder the free passage of the air; P is
a clamping screw, which keeps at its proper level the gallery \(R, R\), which carries the chimney. The next figure (No. 83) shews the burner with the glass chimney and damper. E is the glass-chimney ; \(\mathbf{F}\) is a sheet-iron cylinder, which serves to give it a greater length, and has a small damper D , capable of being turned by a handle for regulating the currents of air; and \(B\) is the pipe which supplies the oil to the wicks. The only risk in using this lamp arises from the liability to occasional derangement of the leathern valves that force the oil by means of clockwork; and several of the lights on the French coast, and more especially the Corduan, have been extinguished by the failure of the lamp for a few minutes, an accident which has never happened, and scarcely can occur with the fountain lamps which illuminate reflectors. To prevent the occurrence of such accidents, and to render their consequences less serious, various precautions have been resorted to. Amongst others, an alarum is attached to the lamp, consisting of a small cup pierced in the bottom, which receives part of the overflowing oil from the wicks, and is capable, when full, of balancing a weight placed at the opposite end of a lever. The moment the machinery stops, the cup ceases to receive the supply of oil, and, the remainder running out at the bottom, the equilibrium of the lever is destroyed, so that it falls and dis-
engages a spring which rings a bell sufficiently loud to waken the keeper should he chance to be asleep. It may justly be questioned whether this alarum would not prove a temptation to the keepers to relax in their watchfulness and fall asleep; and I have, in all the lamps of the dioptric lights on the Scotch coast, adopted the converse mode of causing the bell to cease when the clockwork stops. There is another precaution of more importance, which consists of having always at hand in the light-room a spare lamp, trimmed and adjusted to the height for the focus, which may be substituted for the other in case of accident. It ought to be noticed, however, that it takes about twenty minutes from the time of applying the light to the wicks to bring the flame to its full strength, which, in order to produce its best effect, should stand at the height of nearly four inches ( 10 cm .). The inconveniences attending the great lamp have led to several attempts to improve it ; and, amongst others, M. Delaveleye has proposed to substitute a pump having a metallic piston, in place of the leathern valves, which require constant care, and must be frequently renewed. A lamp was constructed in this manner by M. Lepaute, and tried at Corduan ; but was afterwards discontinued until some of its defects could be remedied. It has lately been much improved by M. Wagner, an ingenious artist, whom M. Fresnel had employed to carry some of his improvements into effect. In the dioptric lights on the Scotch coast, a common lamp, with a large wick, is kept constantly ready for lighting; and, in the event of the sudden extinction of the mechanical lamp by the failure of the valves, it is
only necessary to unscrew and remove its burner, and put the reserve-lamp in its place. The height of this lamp is so arranged, that its flame is in the focus of the lenses, when the lamp is placed on the ring which supports the burner of the mechanical lamp; and as its flame, though not very brilliant, has a considerable volume, it answers the purpose of maintaining the light in a tolerably efficient state for a short time, until the light-keepers have time to repair the valves of the mechanical lamp. Only three occasions for the use of this reserve-lamp have yet occurred.

The most advantageous heights for the flames in dioptric lights are as follow ; -

Inches.
1st Order, \(\quad 10\) to 11 centimètres \(=3.94\) to 4.33
2d Order, \(\quad . \quad 8\) to \(9 \quad \ldots \ldots\)
3d Order, \(\quad . \quad 7\) to \(8 \quad \ldots \ldots\).

Those heights of flame, however, can be obtained only by a careful adjustment of the heights of the wicks and the relative levels of the shoulder of the glass-chimney and the burner, together with a due proportion for the area of the opening of the iron-damper which surmounts it. The wicks must be gradually raised during the first hours of burning to the level of 7 millimètres ( 0.27 inch ) above the burner, a height which they may only very rarely and but slightly exceed. By raising the shoulder of the glass-chimney, the volume of the flame is increased; but, after a certain height is exceeded, the flame, on the other hand, becomes reddish and its brilliancy is diminished. The height of the flame is.
decreased, and it becomes whiter by lowering the chimney. The chimney is lowered or raised by simply turning to the right or to the left the cylindric glassholder in which it rests. In regulating the flame, however, recourse is most frequently had to the use of the damper, by enlarging the opening of which the flame falls and becomes whiter and purer; while by diminishing its aperture, the contrary effect is produced. The area of the opening depends on the inclination of a circular disc capable of turning, vertically through a quadrant, on a slender axle of wire, which is commanded by the light-keeper by means of a fine cord which hangs from it to the table below. When the disc (see fig. 83, p. 95) is in a horizontal plane the chimney is shut; when in a vertical plane, it is open; and each intermediate inclination increases or decreases the aperture.
I need scarcely add, that in order to produce the proper effect of a system of lenses or refractors, the vertical axis of the flame should coincide with their common axis; and it is further necessary, in order to bring the best portion of the flame into a suitable position with reference to the apparatus, that the top of the burner should be quite level, and should stand below the plane of the focus in the following proportions, viz: -
\[
\begin{array}{rlrl}
\text { For 1st order, } 28 \text { millimètres } & =1 \cdot 10 \text { inch. } \\
\ldots & 2 \mathrm{~d} \text { order, } 26 \quad \ldots & =1 \cdot 02 \ldots \\
\ldots .3 \text { order, } 24 \quad \ldots & =0.95 \quad \ldots
\end{array}
\]

For the purpose of placing the lamp in the centre of the apparatus, a plumbet having a sharp point, is suspended in the axis of the apparatus, to indicate, by its
apex, the place for the centre of the burner. The lamp is then raised or lowered as required by means of four adjusting screws at the bottom of its pedestal ; and the top of the burner is made horizontal by a spirit-level, the most convenient form of which is that of the spherical segment, which acts in every azimuth. Its application to this purpose is due, I believe, to M. Letourneau, the successor of M. François in the construction of dioptric apparatus at Paris. This level is shewn in the annexed figure (fig. 84), in which \(a b\) is the brass frame containing the level, and 0 the air-bubble; and \(e\) shews circles of equal altitudes engraved on the glass. Afterthe first application of this level, the adjustment of the burner as to its central position is carefully repeated by means of a centre-gauge (shewn at

Fig. 84.
 fig. 89, p. 106), which is applied to the vertex of each lens, or to many points on the internal surface of the refractors ; and being found correct, the level is again applied to the top of the burner, to detect any deviation from horizontality that may have occurred during the process of adjusting it to the axis.

The lamp is subject to derangement, chiefly from the stiffness of the clack-valves for want of regular cleaning, from bursting of the leathern valves of the oil-box, stiffness of the regulator, or the wearing of the bevelled gearing which gives motion to the connecting-rod that works the valves of the oil-pumps.

The pumps of the lamp should raise, in a given time,
four times the quantity of oil actually consumed in maintaining the flame during that time. Their hourly produce should be,

1b. avoirdupois.
\begin{tabular}{cccc} 
For the lamp with four wicks, & . & . & 6.615 \\
\(\ldots \ldots\) & three wicks, & . & . \\
\(\ldots .410\) \\
\(\ldots .\). & two wicks, . & . & 1.675
\end{tabular}

This surplus of three times what is burned is necessary to prevent the wick from being carbonised too quickly; and it has been found quite sufficient for that purpose. The discharge from the pumps is, of course, regulated by changes in the angle of the fans of the regulator, or in the amount of the moving weight.

In preparing the leathern valves of the pump-box or chamber, shewn in Plate IX., care must be taken that they be neither too flaccid from largeness, nor too tense from smallness; and also that, after being fitted, they draw no air. To remove the old valves and replace them by fresh ones, is a very simple process, more especially when a proper die or mould is used, which at once cuts the kid-leather, of which the valves are formed, to the required size and squeezes them into the proper shape.

The focal point for the lenses and refractors is in the centre of the flame and on the level of its brightest film. The choice of a focus for the zones naturally formed a most important practical consideration in their arrangement For the upper zones, M. Fresnel had adopted a point in the centre of the flame 10 millimètres above the focus of the lenses, so that all the light below that point necessarily falls between the horizon and the lighthouse;
but for the lower zones, it was necessary, owing to their arrangement, for the sake of convenience, in a cylindric form, to adopt a separate focus for each zone, receding from the adjacent one, in the direction of the centre of gravity of that part of the flame which would light each zone. In this manner (fig. 85) the foci of Fig. 85.

the zones move upwards from \(a\) to \(f\) in proportion to the depression of the zones \(a, b, c, d, e, f\), so that the line joining each zone and its focus, must revolve as a radius vector round some point \(O\) between them. The details of this arrangement are shewn in Plate VIII.; and are also given in the Tables of the Catadioptric Zones.

In the are next the land, in fixed lights, a great loss of light ensues from the escape of the rays uselessly in that direction. So far back as 1834 , I suggested the placing a segment of a spherical mirror, with its centre of curvature coincident with \(F\) the focus of the system, so that the luminous pyramid MFM, of which the mirror MM forms the base, might be thrown back through the focal point and finally refracted into such a direction as to contribute to the effect of the lens QA \(q\) in seaward and opposite arc. In the diagram (fig. 86), \(r r\) indicate

Fig. 86.

rays proceeding directly from \(\mathbf{F} ; r^{\prime} r^{\prime}\) rays reflected from MM through F, and finally refracted at QA \(q\); and \(r^{\prime \prime} r^{\prime \prime}\) is the beam compounded of both. In the best glass-silvered mirrors, this accession of light would amount to nearly half of the light incident on them. In such an arrangement, a considerable radius is desirable to decrease the amount of aberration produced by a large flame. In the case of revolving lights of the first order, the radius would, of course, be limited to somewhat less than three feet, which is the focal dis-
tance of the lenses, between which and the focus, the reflecting segment must be placed; but in fixed lights, the lantern is the limit of radius, so that a focal length of five feet ten inches may be obtained. M. François ground some beautiful mirrors of three feet radius, which were afterwards silvered by his successor, M. Letourneau, by some process similar to that patented by Mr Drayton of London, by depositing a thin film of silver from a spirituous solution of the nitrate ;* and that gentleman has successfully completed the construction of reflecting spherical segments \(1200 \mathrm{~mm} \cdot\) square (about 16 superficial feet), to a radius of \(1770 \mathrm{~mm} \cdot(5\) feet 10 inches), which subtend a vertical arc of about \(40^{\circ}\).

The arrangements of the dioptric apparatus in the lightroom will be more fully understood by referring to the Plates.

Plate V. shews an elevation of a revolving dioptric apparatus of the first order; F is the focal point, in which the flame is placed; L, L great annular lenses, forming by their union an octagonal prism, with the lamp in its axis, and projecting, in horizontal beams, the light which they receive from the focus; \(L^{\prime} L^{\prime}\), the upper lenses, forming by their union a frustum of an octagonal pyramid of \(50^{\circ}\) of inclination, and having their foci coinciding in the point F . They parallelise the rays of light which pass over the lenses. M, M are

\footnotetext{
* See notice of a similar process practised about the year 1750 by Mr Rogers of London, ante, p. 2, Part II.
}
plane mirrors, placed above the pyramidal lenses \(L^{\prime} L^{\prime}\), and so inclined as to project the beams reflected from them in planes parallel to the horizon; \(\mathrm{Z}, \mathrm{Z}\) are the lower zones, first used at Skerryvore, in the room of the curved mirrors which were used at Corduan. The lower part of Plate V. shews the moveable framework which carries the lenses and mirrors, and the rollers on which it circulates, with the clockwork which gives motion to the whole.

Plate VII. shews a section of a fixed dioptric light of the first order. \(F\) is the focal point in which the flame is placed; \(R, R\) cylindric refractors, forming by their union a prism of thirty-two sides, or a true cylinder, with the lamp in its axis, and producing a zone of light of equal intensity in every point of the horizon; \(A B C\) and \(A^{\prime} B^{\prime} C^{\prime}\) shew the upper and lower zones which supply the place of the mirrors shewn at \(\mathbf{M}, \mathrm{M}\), in Plate VI.; while DEF shews the cylindric belt as lately improved, with the diagonal joints \(\mathrm{M}, \mathrm{N}, \mathrm{C}\); and \(\mathrm{X}, \mathrm{X}\), represent the diagonal supports for the cupola ABC. This plate, in connection with the enlarged section of the same apparatus at Plate VIII., affords a complete explanation of the arrangement of all the parts.

In Plate VI. M, M are the curved mirrors, ranged in tiers above and below the cylindric refractors, and having their foci coinciding in the point \(F\); the effect of the mirrors increases the power of the light, by collecting and transmitting the rays which would otherwise pass above and below them, without increasing the effect of the light. The elements and dimensions of the mirrors are given in the Plate.

For the purpose of arranging the various parts of the dioptric apparatus in their proper positions, three gauges are employed. The first (fig. 87) is for ascertaining that

Fig. 87.

the lenses \(l, l\), are inclined to each other at the proper horizontal angle, so that their axes shall meet with the proper inclination in F the focus. This is done by means of two arms, whose projecting points \(r, r, r, r\) touch the backs of the lenses, while the graduated are \(c\) indicates the inclination of \(l, l\), to \(l, l\), or the complement of that inclination at \(\mathbf{F}\).
Again, for ascertaining the verticality of the main lenses, or for setting the subsidiary lenses or mirrors shewn in Plate V ., at the required angle of inclination, recourse is had to a clinometer (fig. 88) touching the back of the lens LL by means of studs at \(A, A\), while the spirit-level S indicates, on the graduated limb, the
amount of deviation from the vertical position of the instrument, whether accidental or intentional.

Fig. 88.


Lastly, to test the true position of the lamp itself, with reference to the lenses thus properly arranged, we apply a radius or trainer (fig. 89) which fits into the

Fig. 89.

centre burner at F , while its point A touches the centres of the lenses \(l, l\). At B is a graduated slide, which admits of the trainer being lengthened or shortened to suit various focal distances; and the spirit-level at \(c\) at once corrects any error in the length of the trainer arising from depression or elevation, and also serves to indicate the proper level for the burner which is noticed at page 98 , in speaking of the lamp. The dotted line \(\mathbf{F} \boldsymbol{c}^{\prime} \mathrm{B}^{\prime} \mathrm{A}^{\prime}\) shews the position of the trainer in reference to an adjacent lens.

The elegant apparatus invented by Augustin Fresnel for harbour lights, on the same principle as that just described for sea-lights, is shewn (fig. 90, next page). It consists of thirteen rings of glass of various diameters arranged one above another, in an oval form. The five middle rings have an interior diameter of 11.81 inches ( 30 cm .) and, like those of the larger apparatus, refract equally over the horizontal plane of the focus, the light which they receive from it. The other rings or prisms, of which five are called upper and three lower, are ground and set in such a manner, that they project all the light derived from the focus, in a direction parallel to the other rays, by means of total reflection.

The arrangements of this apparatus, which is distinguished by the addition of external refractors arranged vertically, will be more fully understood by a reference to fig. 90 , which shews its section and plan. F is the focal point in which the flame is placed ; \(r, r\) cylindric refractors, forming, by their union, a cylinder with a lamp in its axis, and producing a zone of light of equal intensity all round the horizon ; and \(x, x\) are catadioptric
prismatic rings acting by total reflection, and giving out
Fig. 90.

zones of light of equal intensity at every point of the horizon. The dotted lines shew the course traversed by the rays of light which proceed from the lamp, and are acted upon by the rings of glass. The letters \(r^{\prime} r^{\prime}\) shew the external prisms, having their axes at right angles to those of the principal bent prisms, composing the refractors at \(r, r\), and revolving around them. This ingenious application of the property of crossed prisms is already described at page 43, Part II.

When this apparatus is employed to light only a part of the horizon, the rings are discontinued on the side next the land, and room is thus obtained for using a common fountain lamp; but when the whole horizon is illuminated, the apparatus must inclose the flame on
every side; and it has in that case been found most convenient to employ the hydrostatic lamp of Thilorier, in which a balance of sulphate of zinc in solution is employed to force the supply of oil to the wicks from the cistern below the flame.

An instrument differing from this small apparatus only in size has lately been introduced into the lighthouses in France, and has also been adopted in Scotland for lights in narrow seas. It has the same number of rings of glass as the small apparatus, and of the same proportional dimensions. Its internal diameter, however, is 500 millimètres (about \(19 \frac{1}{2}\) inches). The following Table gives the elements of the eight prismatic zones (above and below the belt), with the co-ordinates to their centres of curvature, measured from the arris A of the outer or emergent surfaces, in whatever position the zone may lie on the lathe. The dimensions are in millimètres ; but may be easily converted into imperial inches, in the manner described in the Note, p. 76.


The effect of an annular lens, in combination with the great lamp, may be estimated at moderate distances to be nearly equal to that of between 3000 and 4000 Ar gand flames of about an inch diameter; that of a cylindric refractor at about 250 ; and that of a curved mirror may perhaps on an average be assumed at about 10 Ar gand flames.

The dioptric lights used in France are divided into six orders, in relation to their power and range; but in regard to their characteristic appearances, this division does not apply, as, in each of the orders, lights of identically the same character may be found, differing only in the distance at which they can be seen, and in the expense of their maintenance. The six orders may be briefly described as follows:-
\(1 s t\), Lights of the first order having an interior radius or focal distance of 36.22 inches ( 92 cm .), and lighted by a lamp of four concentric wicks, consuming 570 gallons of oil per annum.
\(2 d\), Lights of the second order having an interior radins of 27.55 inches ( 70 cm .), lighted by a lamp of three concentric wicks, consuming 384 gallons of oil per annum.
\(3 d\), Lights of the third order, lighted by a lamp of two concentric wicks, consuming 183 gallons of oil per annum, and having a focal distance of \(19 \cdot 68\) inches ( \(50^{\mathrm{cm} .}\) )
\(4 t h\), Lights of the fourth order, or harbour-lights, having an internal radius of 9.84 inches ( \(25^{\mathrm{cm} .}\) ), and a lamp of two concentric wicks, consuming about 130 gallons of oil per annum.
\(5 t h\), Lights of the fifth order, having a focal distance of \(7 \cdot 28\) inches ( \(185^{\mathrm{cm} .}\) ); and

6th, Lights of the sixth order, having an internal radius of 5.9 inches ( \(15^{\mathrm{cm}}\).), and lighted by a lamp of one wick, or Argand burner, consuming 48 gallons of oil per annum. The more minute subdivisions of orders I consider to be unnecessary.

Those orders are not intended as distinctions; but are characteristic of the power and range of lights, which render them suitable for different localities on the coast, according to the distance at which they can be seen. This division, therefore, is analogous to that which separates our lights into sea-lights, secondary lights, and harbour-lights, terms which are used to designate the power and position, and not the appearance, of the lights to which they are applied.

Each of the above orders is susceptible of certain combinations, which produce various appearances, and constitute the distinctions used for dioptric lights; but the following are those which have been actually employed as the most useful in practice :-

The first order contains, 1st, Lights producing, once in every minute, a great flash, preceded by a smaller one, by the revolution of eight great lenses and eight smaller ones combined with eight mirrors; 2d, Lights flashing once in every half minute, and composed of sixteen half lenses. Those lights may have the subsidiary parts simply catoptric, or dia-catoptric ; and, \(3 d\), Fixed lights, composed of a combination of cylindric pieces, with curved mirrors or catadioptric zones ranged in tiers above and below them.

The second order comprises revolving lights with sixteen or twelve lenses, which make flashes every half
minute; and fixed lights varied by flashes once in every four minutes, an effect which, as already noticed, is produced by the revolution of exterior cylindric pieces.

The third order contains common fixed lights, and fixed lights varied by flashes once in every four minutes.

The fourth order contains simple fixed lights, and fixed lights varied by flashes once in three minutes.

The fifth order has fixed lights varied by flashes once in every three minutes, and fixed lights of the common kind. It has been thought necessary to change the term "fixed lights varied by flashes," for "fixed light with short eclipses," because it has been found that, at certain distances, a momentary eclipse precedes the flash. The sixth order has only fixed lights.

These distinctions depend upon the periods of revolution, rather than upon the characteristic appearance of the light; and therefore seem less calculated to strike the eye of a seaman, than those employed on the coasts of Great Britain and Ireland. In conformity with this system, and in consideration of the great loss of light which results from the application of coloured media, distinctions based upon colour have been generally discarded in the French lights.

The distinctions are, in fact, only four in number, viz.: Fixed; Fixed, varied by flashes;* Revolving, with flashes once a minute; and Revolving, with flashes every half-minute. To those might be added, Revolving, with

\footnotetext{
* The " Feu fixe, varié par des éclats," or "Feu fixe, à courtes éclipses," of Fresnel.
}
bright periods once in two minutes, and perhaps Flashing once in five seconds (as introduced by me at the Little Ross, but I cannot say with such complete success as would induce me to recommend its general adoption). My own experience would also lead me to reject the distinction called "Fixed, varied by flashes," which I do not consider as possessing a marked or efficient character.

Having thus fully described the nature of the catoptric and dioptric modes of illuminating lighthouses, I shall conclude with a comparative view of the merits of both systems, deduced from the experiment made at Gullan-hill during the winters of 1832 and 1833, under the inspection of the Commissioners of Northern Lights. The chief practical result of those trials was, that the light of one of the great annular lenses used in the revolving lights of the first order, was equal to the united effect of eight of the large reflectors employed in the revolving lights on the Scotch coast. It may be said, however, that the dia-catoptric* combination of pyramidal lenses and plane mirrors of Corduan, adds the power of more than two reflectors to the effect of the great lens; but it ought to be remembered that in the French lights, this additional power is used only to compensate for one of the defects of the system by lengthening the duration of the flash, and therefore contributes, if at all, only in a

\footnotetext{
* I use this word to designate the arrangement of pyramidal lenses and plane mirrors, by which the light is first refracted, and then reflected.
}
very indirect manner, to render the light visible to the mariner at a greater distance. M. Fresnel found, that from the smaller divergence of the lens, the eclipses were too long and the bright periods of the revolution too short; and he therefore determined to adopt the horizontal deviation of \(7^{\circ}\) for the upper lenses, with a view to remedy this defect. Assuming, therefore, that it were required to increase the number of reflectors in a revolving light of three sides, so as to render it equal in power to a dioptric revolving light of the first order, it would be necessary to place eight reflectors on each face, so that the greatest number of reflectors required for this purpose may be taken at twenty-four. M. Fresnel has stated the expenditure of oil in the lamp of four concentric wicks at 750 grammes of colza oil per hour ; and it is found by experience at the Isle of May and Inchkeith, that the quantity of spermaceti oil consumed by the great lamp, is equal to that burned by from fourteen to sixteen of the Argand lamps used in the Scotch lights. It therefore follows, that, by dioptric means, the consumption of oil necessary for between fourteen and sixteen reflectors, will produce a light as powerful as that which would require the oil of twenty-four reflectors in the catoptric system of Scotland; and, consequently, that there is an excess of oil equal to that consumed by ten reflectors, or 400 gallons in the year, against the Scotch system. But in order fully to compare the economy of producing two revolving lights of equal power by those two methods, it will be necessary to take into the calculation the interest of the first outlay in establishing them.

The expense of fitting up a revolving light with twenty-four reflectors, ranged on three faces, may be estimated at \(£ 1298\), and the annual maintenance, including the interest of the first cost of the apparatus, may be calculated at \(£ 418,8 s .4 \mathrm{~d}\). The fitting up a revolving light with eight lenses and the dia-catoptric accessory apparatus, may be estimated at \(£ 1459\), and the annual maintenance at \(£ 354,10 \mathrm{~s} .4 \mathrm{~d}\). It therefore follows, that to establish and afterwards maintain a catoptric light of the kind called revolving white, with a frame of three faces, each equal in power to a face of the dioptric light of Corduan, an annual outlay of \(£ 63,18 \mathrm{~s}\). more would be required for the reflecting light than for the lens light; while for a light of the kind called revolving red and white, whose frame has four faces, at least thirty-six reflectors would be required in order to make the light even approach an equality to that of Corduan ; and the catoptric light would in that case cost \(£ 225\) more than the dioptric light.

The effect produced by burning an equal quantity of oil in revolving lights on either system, may be estimated as follows:-In a revolving light, like that of Skerryvore, having eight sides, each lighting with its greatest power a horizontal sector of \(4^{\circ}\), we have \(32^{\circ}\) (or units) of the horizon illuminated with the full power of 3200 Argand flames, and consequently an aggregate effect of 102,400 flames, produced by burning the oil required for sixteen reflectors; while in a catoptric apparatus, like that of the old light at Inchkeith, having seven sides of one reflector, each lighting with its greatest power a sector of \(4^{\circ} \cdot 25^{\prime}\), we have nearly \(31^{\circ}\)
(or units) of the horizon illuminated with the full power of 400 Argand flames, and consequently an aggregate effect of 12,400 flames as the result of burning the oil required for seven reflectors. Hence, the effect of burning the same quantity of oil in revolving lights on either system, will be represented respectively by \(\frac{16}{7} 12,400=\) 28,343 for the catoptric, contrasted with 102,400 for the dioptric light; or, in other words, revolving lights on the dioptric principle, use the oil more economically than those on the catoptric plan, nearly in the ratio of \(3 \cdot 6\) to 1 .

Let us now speak of fixed lights, to which the dioptric method is peculiarly well adapted. The effect produced by the consumption of a gallon of oil in a fixed light, with twenty-six reflectors, which is the smallest number that can be properly employed, may be estimated as follows :-The mean effect of the light spread over the horizontal sector, subtended by one reflector, as deduced from measurements made at each horizontal degree, by the method of shadows, is equal to 174 unassisted Argand burners. If, then, this quantity be multiplied by 360 degrees, we shall obtain an aggregate effect of 62,640 , which, divided by 1040 (the number of gallons burned during a year in twenty-six reflectors), would give 60 Argand flames for the effect of the light maintained throughout the year by the combustion of a gallon of oil. On the other hand, the power of a catadioptric light of the first order, like that lately established at Girdleness, may be estimated thus:-The mean effect of the light produced by the joint effect of both the diop-
tric and catadioptric parts of a fixed light apparatus, may be valued at 450 Argand flames, which, multiplied by 360 degrees, gives an aggregate of 162,000 ; and if this quantity be divided by 570 (the number of gallons burned by the great flame in a year), we shall have about 284 Argand flames for the effect of the light produced by the combustion of a gallon of oil. It would thus appear that in fixed lights, the French apparatus, as lately improved, produces, as the average effect of the combustion of the same quantity of oil over the whole horizon, upwards of four times the amount of light that is obtained by the catoptric mode.

But the great superiority of the dioptric method chiefly rests upon its perfect fulfilment of an important condition required in a fixed light, by distributing the rays equally in every point of the horizon. In the event of the whole horizon not requiring to be illuminated, the dioptric light would lose a part of its superiority in economy, and when half the horizon only is lighted, it would be more expensive than the reflected light ; but the greater power and more equal distribution of the light, may be considered of so great importance, as far to outweigh the difference of expense. In the latter case, too, an additional power, as already noticed (p. 102), can be given to the dioptric light, by placing at the landward side of the lightroom, spherical mirrors with their centres in the focus of the refracting apparatus.* The luminous cones, or pyramids, of which such

\footnotetext{
* A similar arrangement can also be made in revolving lights by making the radius of the mirrors somewhat less than that of
}
reflectors would form the bases, instead of passing off uselessly to the land, would thus be thrown back through the focal point, and finally refracted, so as to increase the effect of the light seaward, by nearly one-third of the light which would otherwise be lost.

The expense of establishing a fixed light composed of twenty-six reflectors, may be estimated at \(£ 950\), and its annual maintenance, including interest on the first cost of the apparatus, may be reckoned at \(£ 425,10 \mathrm{~s}\).: and the expense of fitting up a fixed light on the dioptric principle with catadioptric zones is \(£ 1511\), while its annual maintenance may be taken at \(£ 285,6 \mathrm{~s} .4 \mathrm{~d}\). It thus appears that the annual expenditure of the dioptric fixed light is \(£ 140,3 \mathrm{~s} .8 \mathrm{~d}\). less than that of a fixed light composed of twenty-six reflectors; while the average effect, equally diffused over the horizon, is four times greater.

The comparative views already given of the catoptric and dioptric modes of illuminating lighthouses, demonstrate that the latter produces more powerful lights by the combustion of the same quantity of oil; while it is obvious that the catoptric system insures a more certain exhibition of the light, from the fountain-lamps being less liable to derangement than the mechanical lamps
the inscribed circle of the octagon bounded by the lenses, so that they may circulate freely round the backs of the mirrors. The shortness of the radius of the reflecting surface would, of course, increase the divergence of the beam of light refracted through the lenses, as the flame would, in this case, subtend a greater angle at the face of the mirrors.
used in dioptric lights. The balance, therefore, of real advantages or disadvantages, and, consequently, the propriety of adopting the one or the other system, involves a mixed question, not susceptible of a very precise solution, and leaving room for different decisions, according to the value which may be set upon obtaining a cheaper and better light, on the one hand, as contrasted, on the other, with less certainty in its exhibition. Experience, however, goes far to shew that, in practice, the risk of extinction of the lamp in dioptric lights is very small.
A few general considerations, serving briefly to recapitulate the arguments for and against the two systems, may not be out of place. And, first, regarding the fitness of dioptric instruments for revolving lights, it appears from the details above given,-
\(1 s t\), That by placing eight reflectors on each face of a revolving frame, a light may be obtained as brilliant as that derived from the great annular lens; and that, in the case of a frame of three sides, the excess of expense by the reflecting mode, would be \(£ 63,18 \mathrm{~s}\).; and in the case of a frame of four sides, the excess would amount to \(£ 225\).
\(2 d\), That for burning oil economically in revolving lighthouses, which illuminate every point of the horizon successively, the lens is more advantageous than the reflector in the ratio of 3.6 to 1 .
\(3 d\), That the divergence of the rays from the lens being less than from the reflector, it becomes difficult to produce, by lenses, the appearance which characterises the catoptric revolving lights, already so well known to

British mariners; and any change of existing lights which would, of course, affect their appearance, must, therefore, involve some practical objections, which do not at all apply to the case of new lights.

4th, That the uncertainty in the management of the lamp renders it more difficult to maintain the revolving dioptric lights without risk of extinction, an accident which has several times occurred at Corduan and other lighthouses both in France and elsewhere. A more extended experience, however, has tended to moderate any fears on this head.
\(5 t h\), That the extinction of one lamp in a revolving catoptric light is not only less probable, but leads to much less serious consequences than the extinction of the single lamp in a dioptric light; because, in the first case, the evil is limited to diminishing the power of one face by an eighth part; whilst, in the second, the whole horizon is totally deprived of light. The extinction of a lamp, therefore, in a dioptric light, leads to evils which may be considered very great in comparison with the consequences which attend the same accident in a catoptric light.

In comparing the fixed dioptric, and the fixed catoptric apparatus, the results may be summed up under the following heads:-
\(1 s t\), It is impossible, by means of any practicable combination of paraboloidal reflectors, to distribute round the horizon a zone of light of exactly equal intensity; while this may be easily effected by dioptric means, in the manner already described. In other words,
the qualities required in fixed lights cannot be so fully obtained by reflectors as by refractors.
\(2 d\), The average light produced in every azimuth by burning one gallon of oil in Argand lamps, with reflectors, is only about one-fourth of that produced by burning the same quantity in the dioptric apparatus; and the annual expenditure is \(£ 140,3 \mathrm{~s}\). 8d. less for the entire dioptric light than for the catoptric light.

3 l , The characteristic appearance of the fixed reflecting light in any one azimuth would not be changed by the adoption of the dioptric method, although its increased mean power would render it visible at a greater distance in every direction.

4th, From the equal distribution of the rays, the dioptric light would be observed at equal distances in every point of the horizon ; an effect which cannot be fully attained by any practicable combination of paraboloidal reflectors.

5th, The inconveniences arising from the uncertainty which attends the use of the mechanical lamp, are not perhaps so much felt in a fixed as in a revolving light; because the greater simplicity of the apparatus admits of easier access to it, in case of accident.

6th, But the extinction of a lamp in a catoptric light, leaves only one twenty-sixth part of the horizon without the benefit of the light, and the chance of accident arising to vessels from it, may, therefore, be considered as incalculably less than the danger resulting from the extinction of the single lamp of the dioptric light, which deprives the whole horizon of light.

7th, There is also, in certain situations, a risk arising from irregularity in the distances at which the same fixed catoptric light can be seen in the different azimuths. This defect, of course, does not exist in the dioptric light.

There can be little doubt, that the more fully the system of Fresnel is understood, the more certainly will it be preferred to the catoptric system of illuminating lighthouses, at least in those countries where this important branch of administration is conducted with the care and solicitude which it deserves. It must not, however, be imagined, that there are no circumstances in which the catoptric system is not absolutely preferable to illumination by means of lenses. We have hitherto attended only to horizontal divergence and its effects, and this is unquestionably the more important view; but the consideration of vertical divergence must not be altogether overlooked. Now, while it is obvious that vertical divergence, at least above the horizon, involves a total loss of the light which escapes uselessly upwards into space (in which respect the reflectors are much less advantageous), it is no less true, that if the sheet of light which reaches the most distant horizon of the lighthouse, however brilliant, were as tlin as the absence of all vertical divergence would imply, it would be practically useless; and some measure of dispersion in the arc below the horizon is therefore absolutely indispensable to constitute a really useful light. In the reflector, the greatest vertical divergence below the horizontal plane of the focus is \(16^{\circ} 8^{\prime}\), and that of the lens is about \(4^{\circ} 30^{\prime}\). Let us consider for a moment the bearing of those
facts upon the application of the two modes of illumination to special circumstances. The powerful beam of light transmitted by the lens, peculiarly fits that instrument for the great sea-lights which are intended to warn the mariner of his approach to a distant coast which he first makes on an over-sea voyage; and the deficiency of its divergence, whether horizontal or vertical, is not practically felt as an inconvenience in lights of that character, which seldom require to serve the double purpose of being visible at a great distance, and at the same time of acting as guides for danger near the shore. For such purposes, the lens applies the light much more advantageously as well as more economically than the reflector; because, while the duration of its least divergent beam is nearly equal to that of the reflector, it is eight times more powerful. A revolving system of eight lenses illuminates an horizontal arc of \(32^{\circ}\) with this bright beam. The reflector, on the other hand, spreads the light over a larger are of the horizon; and, while its least divergent beam is much less powerful than that of the lens, the light which is shed over its extreme arc is so feeble as to be practically of no use in lights of extensive range, even during clear weather. When a lighthouse is placed on a very high headland, however, the deficiency of divergence in the vertical direction is often found to be productive of some practical inconvenience; but this defect may be partially remedied by giving to the lenses a slight inclination outwards from the vertical plane of the focus, so as to cause the most brilliant portion of the emergent bam to reach the visible horizon which is due to the
height of the lantern. It may be observed, also, that a lantern at the height of 150 feet, which (taking into account the common height of the observer's eye at sea) commands a range of upwards of 20 English miles, is sufficient for all the ordinary purposes of the navigator, and that the intermediate space is practically easily illuminated, even to within a mile of the lighthouse, by means of a slight inclination of the subsidiary mirrors, even where the light from the principal part of the apparatus passes over the seaman's head. For the purpose of leading lights, in narrow channels, on the other hand, and for the illumination of certain narrow seas, there can be no doubt that reflectors are much more suitable and conventent. In such cases, the amount of vertical divergence below the horizon, forms an important element in the question, because it is absolutely necessary that the mariner should keep sight of the lights even when he is very near them; while there is not the same call for a very powerful beam which exists in the case of sea-lights. Yet even in narrow seas, where low towers, corresponding to the extent of the range of the light, are adopted, but where it is, at the same time, needful to illuminate the whole or the greater part of the horizon, the use of dioptric instruments will be found almost unavoidable, especially in fixed lights, as well from their equalizing the distribution of the light in every azimuth, as from their much greater economy in situations where a large annual expenditure would often be disproportionate to the revenue at disposal. In such places, where certain peculiarities of the situation require the combination of a light equally diffused
over the greater portion of the horizon, along with a greater vertical divergence in certain azimuths, than dioptric instruments afford, I have found it convenient and economical to add to the fixed refracting apparatus a single paraboloidal reflector, in order to produce the desired effect, instead of adapting the whole to the more expensive plan for the sake of meeting the wants of a single narrow sector of its range. In other cases, where the whole horizon is to be illuminated, and great vertical divergence is at the same time desirable, a slight elevation of the burner, at the expense, no doubt, of a small loss of light, is sometimes resorted to, and is found to produce, with good effect, the requisite depression of the emergent rays.

In certain situations, where a great range, and, consequently, a powerful light must be combined with tolerably powerful illumination in the immediate vicinity of the lighthouse, we might, perhaps, advantageously adopt a variation of the form and dimensions of the mirrors employed, so as to resemble those formerly used at the Tour de Corduan, which were of considerably larger surface and longer focal distance than those which are used in Britain. If such a form were adopted, the power of the light for the purpose of the distant range would be increased ; and I would propose to compensate for the deficiency of divergence consequent on a long focal distance, by placing a second burner in some position between the parameter and the vertex, and slightly elevated above the axis of the instrument, so as to throw the greater portion of the beam resulting from this second burner below the horizontal plane of the focus.

Such an expedient is no doubt somewhat clumsy, and would at the same time involve the consumption of twice the quantity of oil used in an ordinary catoptric light; but I can still conceive it to be preferable, in certain situations, to the use of lenses alone.

Thus it appears that we must not too absolutely conclude against one, or in favour of the other mode of illumination for lighthouses; but, as in every other department of the arts, we shall find the necessity of patiently weighing all the circumstances of each particular case that comes before us, before selecting that instrument, or combination of instruments, which appears most suitable.

The mode of distinguishing lights in the system of Fresnel, depends more upon their magnitude and the measured interval of the time of their revolution, than upon their appearance; and no other very marked distinctions, except Fixed and Revolving, have been successfully attempted in France. As above stated, I consider the distinction of the fixed light varied by fashes, to possess an appearance too slightly differing from that of a revolving light, to admit of its being safely adopted in situations where revolving lights are near. The trial which I made at the Little Ross, in the Solway Frith, of producing, by means of lenses, a light flashing once in five seconds of time, although successful so far as mere distinction is concerned, has several practical defects, arising from the shortness of the duration of the flashes, compared with the powerful effect of the fixed part of the apparatus, which I consider sufficient to prevent its adoption in future, especially considering that
a much more marked appearance can be produced by means of reflectors, as has been done at the Buchanness in Aberdeenshire, and the Rhinns of Islay in Argyllshire. Coloured media have never, so far as I know, been applied to dioptric apparatus, except in the case of the Maplin Light at the mouth of the Thames, and Cromarty Point Light at the entrance to the Cromarty Frith, Nosshead in Caithness, and Ship Rock of Sanda in Argyleshire, but in all those instances successfully. In the case of the fixed light at Sanda, in particular, I would observe that it is well seen at the distance of 16 nautic miles, and occasionally observed even so far off as 22 nautic miles. The enormous loss of light, however, amounting to no less than 0.80 of the whole incident rays, forms a great bar to the adoption of colour as a distinction; and any means which could tend to lessen that absorption, and at the same time produce the characteristic appearance, would be most valuable. I have tried some glasses of a pink tinge, prepared by M. Letourneau of Paris, in which the absorption does not exceed 0.57 of the incident rays;- but the appearance of the light, at a distance, is much less marked than that produced by the glasses used in Britain.* Such deficiency of characteristic colour might lead to serious consequences, as the transmission of white rays, through a hazy atmosphere, too often produces, by absorption, a reddish tinge of the light, for which the less marked appearance given by the paler media might be easily mistaken. This colouring power

\footnotetext{
* See page 108, Part I.
}
of absorption is so well known, that red lights are seldom used except in direct contrast with white ones; but, on a coast so thickly studded with lighthouses as that of Britain, the number of distinctions is insufficient to supply all our wants, so that we are sometimes reluctantly compelled to adopt a single red light in some situation of lesser importance, or which, from some local circumstances and the appearance of the lights which must be seen by the mariner before passing it, is not likely to be mistaken for any other. The great loss of light by coloured media causes the red beam, in a revolving light, to be seen at a shorter distance than the white; and it is conceivable that, in certain circumstances, this might lead the mariner to mistake a red and white light for a white light revolving at half the velocity. Such a mistake might perhaps prove dangerous; but the lights are generally so situated that there is ample time for the mariner, after first discovering the red light, and thus correcting any mistake, to shape his course accordingly. All other coloured media except red have been found useless as distinctions for any lights of extensive range, and fail to be efficient, owing to the necessity of absorbing almost all the light before a marked appearance can be obtained. In a few pier or ferry lights, green and blue media have been tried, and found available at the distance of a few cables' lengths.*

It seems to be a natural consequence of the physical

\footnotetext{
* In some late experiments which I made with very powerful instruments, green lights were visible, in very clear weather, at the distance of 7 miles. The blue could only once be seen, with great difficulty, at 5 miles.
}
distribution of light, that fixed lights, which illuminate the whole horizon, should be less powerful than revolving lights which have their effect concentrated within narrow sectors of the horizon. Any attempt to increase the power of fixed lights is, therefore, worthy of attention ; and when the late Captain Basil Hall proposed a plan for effecting this object, it received, as it deserved, the full consideration of the Scotch Lighthouse Board, who authorized me to repeat Captain Hall's experiments, and verify his results by observations made at a considerable distance.

The familiar experiment of whirling a burning stick quickly round the head, so as to produce a ribbon of light, proves the possibility of causing a continuous impression on the retina by intermittent images succeeding each other with a certain rapidity. From the moderate velocity at which this continuity of impression is obtained, we should be warranted in concluding, a priori, that the time required to make an impression on the retina is considerably less than the duration of the impression itself; for the continuity of effect must, of course, be caused by fresh impulses succeeding each other before the preceding ones have entirely faded. If it were otherwise, and the time required to make the impression were equal to the duration of the sensation, it would obviously be impossible to obtain a series of impulses so close or continuous in their effects as to run into and overlap each other, and thus throw out the intervals of darkness; because the same velocity which would tend to shorten the dark intervals, would also curtail the bright flashes, and thus prevent their acting on the eye long enough to cause an impression. Accordingly, we
find that the duration of an impression is in reality much greater than the time required for producing the effect on the retina. It is stated by Professor Wheatstone, in the London Transactions for 1834, that only about one millionth part of a second is required for making a distinct impression on the eye; and it appears, from a statement made by Lamé, at p. 425 of his Cours de Physique, that M. Plateau found that an impression on the retina preserved its intensity unabated during one hundredth of a second, so that, however small those times may be in themselves, the one is yet 10,000 greater than the other.

It has been ascertained by direct experiment,* that the eye can receive a fresh impression before the preceding one has faded; and, indeed, if this were impossible, absolute continuity of impression from any succession of impulses, however rapid, would seem to be unattainable; and the approach to perfect continuity would be inversely as the time required to make an impression.

From the property which bright bodies passing rapidly before the eye possess of communicating a continuous impression to the sense of sight, the late Captain Basil Hall conceived the idea, not merely of obtaining all the effects of a fixed light, by causing a system of lenses to revolve with such a velocity as to produce a continuous impression, but, at the same time, of obtaining a much more brilliant appearance, by the compensating influence of the bright flashes, which he expected

\footnotetext{
* Lamé, Cours de Physique, p. 424. "L’impression peut subsister encore lorsque la suivante a lieu."
}
would produce impulses sufficiently powerful and durable to make the deficiency of light in the dark spaces almost imperceptible. The mean effect of the whole series of changes would, he imagined, be thus greatly superior to that which can be obtained from the same quantity of light equally distributed, as in fixed lights, over the whole horizon. Now this expectation, if it be considered solely in reference to the physical distribution of the light, involves various difficulties. The quantity of light subjected to instrumental action is the same whether we employ the refracting zones at present used in fixed dioptric lights, or attempt to obtain continuity of effect by the rapid revolution of lenses; and the only difference in the action of those two arrangements is this, that while the zones distribute the light equally over the whole horizon, or rather do not interfere with its natural horizontal distribution, the effect of the proposed method is to collect the light into pencils, which are made to revolve with such rapidity, that the impression from each pencil succeeds the preceding one in time to prevent a sensible occurrence of darkness. To expect that the mean effect of the light, so applied, should be greater than when it is left to its natural horizontal divergence, certainly appears at first to involve something approaching to a contradiction of physical laws. In both cases, the same quantity of light is acted upon by the instrument; and in either case, any one observer will receive an impression similar and equal to that received by any other stationed at a different part of the horizon; so that, unless we imagine that there is some loss of light peculiar to one of the methods, we are shut up, in the physical view of the question, to the
conclusion, that the impressions received by each class of observers must be of equal intensity. In other words, the same quantity of light is by both methods employed to convey a continuous impression to the senses of spectators in every direction, and in both methods equality of distribution is effected, since it does not at all consist with our hypothesis, that any one observer in the same class should receive more or less than his equal share of the light. Then, as to the probability of the loss of light, it seems natural to expect that this should occur in connection with the revolving system, because the velocity is an extraneous circumstance, by no means necessary to an equal distribution of the light, which can, as we already know, be more naturally, and at the same time perfectly attained by the use of the zones.

On the other hand, it must not be forgotten, that although the effect of both methods is to give each part of the horizon an equal share of light, there is yet this difference between them, that while the light from the zones is equally intense at every instant of time, that evolved by the rapidly-circulating lenses is constantly passing through every phase between total darkness and the brightest flash of the lens; and this difference, taken in connection with some curious physiological observations regarding the sensibility of the retina, gives considerable countenance to the expectation on which Captain Hall's ingenious expedient is based. The fact which has already been noticed, and which the beautiful experiments of M. Plateau and Professor Wheatstone have of late rendered more precise, that the duration of an impression on the retina is not only appreciable, but is much greater than the time required to
cause it, seems to encourage us in expecting, that while the velocity required to produce continuity of effect would not be found so great as to interfere with the formation of a full impression, the duration of the impulse from each flash would remain unaltered, and the dark intervals which do not excite the retina would, at the same time, be shortened; and that, therefore, we might in this manner obtain an effect on the senses exceeding the brilliancy of a steady light distributed equally in every direction by the ordinary method. Some persons, indeed, who have speculated on this subject, seem even to be of opinion, that, so far from the whole effect of the series of continuous impressions being weakened by a blending of the dark with the bright intervals, the eye would in reality be stimulated by the contrast of light and darkness, so as thereby to receive a more complete and durable impulse from the light. It is obvious, however, that this question regarding the probable effect to be anticipated from a revolution so rapid as to cause a continuous impression, could only have been satisfactorily answered by an appeal to experiment.

In experimenting on this subject, I used the apparatus formerly employed by Captain Hall. It consisted of an octagonal frame, which carried eight of the discs that compose the central part of Fresnel's compound lens, and was susceptible of being revolved slowly or quickly at pleasure, by means of a crank-handle and some intermediate gearing. The experiments were nearly identical with those made by Captain Hall, who contrasted the effect of a single lens at rest, or moving very slowly, with that produced by the eight lenses, revolving with such velocity as to cause an apparently
continuous impression on the eye. To this experiment I added that of comparing the beam thrown out by the central portion of a cylindric refractor, such as is used at the fixed light of the Isle of May, with the continuous impression obtained by the rapid revolution of the lenses. Captain Hall made all his comparisons at the short distance of 100 yards; and, in order to obtain some measure of the intensity, he viewed the lights through plates of coloured glass until the luminous discs became invisible to the eye. I repeated those experiments at Gullan, under similar circumstances, but with very different results. I shall not, however, enter upon the discussion of those differences here, although they are susceptible of explanation, and are corroborative of the conclusions at which I arrived, by comparing the lights from a distance of 14 miles; but shall briefly notice the more important.results which were obtained by the distant view. They are as follow :-
1. The flash of the lens revolving slowly was very much larger than that of the rapidly revolving series; and this decrease of size in the luminous object presented to the eye became more marked as the rate of revolution was accelerated, so that, at the velocity of eight or ten flashes in a second, the naked eye could hardly detect it, and only a few of the observers saw it; while the steady light from the fixed refractor was distinctly visible.
2. There was also a marked falling off in the brilliancy of the rapid flashes as compared with that of the slow ones; but this effect was by no means so striking as the decrease of volume.
3. Continuity of impression was not attained at the
rate of five flashes in a second, but each flash appeared to be distinctly separated by an interval of darkness; and even when the nearest approach to continuity was made, by the recurrence of eight or ten flashes in a second, the light still presented a twinkling appearance, which was well contrasted with the steady and unchanging effect of the cylindric refractor.
4. The light of the cylindric refractor was, as already stated, steady and unchanging, and of much larger volume than the rapidly revolving flashes. It did not, however, appear so brilliant as the flashes of the quickly revolving lenses, more especially at the lower rate of five flashes in a second.
5. When viewed through a telescope, the difference of volume between the light of a cylindric refractor and that produced by the lenses at their greatest velocity was very striking. The former presented a large diffuse object of inferior brilliancy, while the latter exhibited a sharp pin-point of brilliant light.

Upon a careful consideration of these facts, it appears warrantable to draw the following general conclusions :-
1. That our expectations as to the effects of light, when distributed according to the law of its natural horizontal divergence, are supported by observed facts as to the visibility of such lights, contrasted with those whose continuity of effect is produced by collecting the whole light into bright pencils, and causing them to revolve with great velocity.
2. It appears that this deficiency of visibility seems to be chiefly due to a want of volume in the luminous object, and also, although in a less degree, to a loss of
intensity, both of which defects appear to increase in proportion as the motion of the luminous object is accelerated.
3. That this deficiency of volume is the most remarkable optical phenomenon connected with the rapid motion of luminous bodies, and that it appears to be directly proportional to the velocity of their passage over the eye.
4. That there is reason to suspect that the visibility of distant light depends on the volume of the impression in a greater degree than has perhaps been generally imagined.
5. That, as the size and intensity of the radiants causing these various impressions to a distant observer were the same, the volume of the light, and, consequently, coeteris paribus, its visibility, are, within certain limits, proportionate to the time during which the object is present to the eye.

Such appear to be the general conclusions which those experiments warrant us in drawing; and the practical result, in so far as lighthouses are concerned, is sufficient to discourage us from attempting to improve the visibility of fixed lights in the manner proposed by Captain Hall, even supposing the practical diffieulties connected with the great centrifugal force generated by the rapid revolution of the lenses to be less than they really are.

The decrease in the volume of the luminous object caused by the rapid motion of the lights is interesting, from its apparent connection with the curious phenomenon of irradiation. When luminous bodies, such as
the lights of distant lamps, are seen by night, they appear much larger than they would do by day; and this effect is said to be produced by irradiation. M. Plateau, in his elaborate essay on this subject, after a careful examination of all the theories of irradiation, states it to be his opinion, that the most probable mode of accounting for the various observed phenomena of irradiation is to suppose, that, in the case of a night-view, the excitement caused by light is propagated over the retina beyond the limits of the day-image of the object, owing to the increased stimulus produced by the contrast of light and darkness ; and he also lays it down as a law, confirmed by numerous experiments, that irradiation increases with the duration of the observation. It appears, therefore, not unreasonable to conjecture, that the deficiency of volume observed during the rapid revolution of the lenses may have been caused by the light being present to the eye so short a time, that the retina was not stimulated in a degree sufficient to produce the amount of irradiation required for causing a large visual object. When, indeed, the statement of M. Plateau, that irradiation is proportional to the duration of the observation, is taken in connection with the observed fact, that the volume of the light decreased as the motion of the lenses was accelerated, it seems almost impossible to avoid connecting together the two phenomena as cause and effect.

Before leaving this part of the subject, I will call attention to some late plans for combining dioptric and catoptric apparatus, the object of which is to subject to the corrective action of instruments, a greater proportion of
the luminous sphere than it has yet been found practicable to do, especially in revolving lights. Reflectors act chiefly on the posterior portion of the flame, and generally receive about \(\frac{1}{1} \frac{2}{7}\) ths of the whole luminous sphere; while a series of dioptric instruments can only affect an anterior zone, amounting to about \(\frac{8}{5}\) ths of the whole light which is emitted by the lamp. Certain deductions due to the form of the lower part of the burners, and to the loss of light at reflection, which is not less than one-half of the incident light, as well as to that, by refraction, through the lens (which, however, cannot exceed one-tenth of the incident light), will reduce those numbers from \(\frac{18}{\frac{9}{7}}\) ths to \(\frac{1}{3} d\), and from \(\frac{9}{5}\) ths to \(\frac{1}{1}^{\frac{3}{0}}\) ths, thus making the ratio of the proportion of the whole flame actuallygiven forth by the reflectors to the amount bylenses equal to that of 10 to 9 . In fixed lights, on Fresnel's system, we have already seen that nearly the whole of the available light is turned to a useful purpose, by means of the curved mirrors or catadioptric zones, which are added to increase the effect of the central dioptric belt; and, in revolving lights, an approximation to a similar result is obtained by the addition of the dia-catoptric combination of pyramidal lenses and plane mirrors, placed above the great lenses. Catoptric lights, however, to which such auxiliary arrangements are inapplicable, had still the great disadvantage of leaving the anterior cone of light to pass off in the useless state of naturally divergent light; and any thing calculated to increase the power of that class of lights, without altering that simplicity and security of the burners employed in them, which renders them so suitablefor remote situations
in the colonies, deserves careful attention. It will be remembered that the proposal of Mr Barlow for effecting this object, has already been noticed; and it is needless now to do more than remind the reader, that the practical disadvantage of the great aberration in the path of the rays reflected from the subsidiary hemispherical mirror, which must necessarily be of very small dimensions, together with the great loss of light by the second reflection, must go far to neutralize the effect of Mr Barlow's plan. A combination of dioptric and catoptric instruments, intended to produce a similar effect, has been proposed by Mr Alexander Gordon, and is described at page 385 of the tenth volume of the Civil Engineers' and Architects' Journal. It consists of a paraboloïdal mirror, of a very short focal distance, with some of the outer zones of one of Fresnel's smaller lenses in front of it. The zones are intended to refract some of the rays that escape past the edges of the mirror, while the pencil of light reflected from the mirror itself is supposed to pass through the circular space which is generally occupied by the central portion of the lens. This arrangement is a step in the right direction, only in so far as it implies the union of the two modes of illumination; but, as it is by no means skilfully designed, it is liable to several palpable objections.

1 stly, The actual gain of light has been greatly overrated by the writer in the Journal, who expects to turn \(\frac{97}{3} \frac{7}{8}\) th of the whole light to a useful account ; but so great a gain of light can never consist with the form and positiont of the lower part of the flame.
\(2 d l y\), Upwards of \({ }_{2}^{2} \frac{2}{8}\) ths of the estimated quantity
would be intercepted by the paraboloïd alone, and little more than \(2^{2}\) the by the rings of the lens, an addition far too insignificant to warrant the adoption of so expensive an appendage to the reflector.
\(3 d l y\), The great aberration of the rays reflected by the conoid behind the paramcter, and its small reflecting surface, must render it practically useless; and, perhaps, nearly one-half of the whole light would thus be lost to the mariner. The accurate formation of a paraboloïd of such depth would also be difficult; and, considering the practical inutility of the conoid behind the parameter, would seem to be a misapplication of labour.

4thly, The union of such an instrument with the lenticular zones in front, which require that the pencil of parallel rays should be reflected with the greatest accuracy, so as to enable them to pass through the circular space bounded by the zones, is an obvious misapplication of a paraboloid with a short focal distance, to a purpose for which it is singularly unsuitable.

5 thly, Mirrors somewhat of the same form werc in use at Scilly Lighthouse, and were long ago discarded as disadvantageous, at the suggestion of the late cminent Captain Huddart.
6 thly, The outer zoncs, which form the least efficient, and at the same time the most expensive portion of the compound lens have been preferred to the central portion of that instrument; and by this means the anterior cone of rays is at the same time lost.

There can be little doubt that it is by the union of reflection and refraction alonc, that any thing like an approach to a complete application of the whole light cau be reasonably expected; and it was this considera-
tion which led me, in 1834, to suggest the addition of spherical mirrors to dioptric lights, which illuminate only one-half of the horizon.

The various forms of apparatus proposed by Mr Thomas Stevenson for increasing the power of reflectors, and for otherwise improving revolving lights with moderately-sized burners, accordingly agree in employing both modes; and, while they are free from theoretical objections, they, at the same time, subject the whole of the available light to the corrective action of the instrument. From this common property they have been called Holophotal,* although differing widely in the nature of their action and the amount of light transmitted by them.

In the first arrangement, which is shewn in figs. 91 and 92 (of which the first is a section, in the line of the axis, and the latter is a front view), the flame is supposed

Fig. 91.


Fig. 92.

to be divided into three parts, whereof the anterior cone is made parallel by the lens at \(e\); and the remaining zone,

\footnotetext{
* \(0^{\circ} \lambda_{0}\), entire- \(\varphi \dot{\omega} \varsigma\), light.
}
by the paraboloïdal surfaces between \(n o\) and \(p p\); while the posterior hemisphere is received on the hemispherical mirror \(n m o\), and by it sent back through the focus \(f\), whence, passing onward, it is in part refracted by the lens \(e\), and in part reflected by the paraboloidal surfaces between \(n o\) and \(p p\), and thus finally emerges horizontally, in union with the light from the anterior hemisphere. The hemispherical mirror, of course, occupies the place of the parabolic conoild which is cut off behind the parameter; and its dimensions may be increased at pleasure, so as to reduce the aberration of the reflected rays. It applies equally to fixed as to revolving lights, its object being in both cases the same, viz., to reassemble the posterior hemisphore of rays in the focus \(f\).

In generating the apparatus for the revolving light, the section, at fig. 91 , is supposed to revolve round the horizontal axis efm; but, for fixed lights, the apparatus is generated by the revolution of the parabola in a horizontal path round a vertical axis in the parameter \(n f o\), as in the case of M. Bordier Marcet's harbour light (see page 114, Part I.), while the section of the lens also revolves round the same vertical axis, and thus generates the dioptric belt of Fresnel's catadioptric apparatus (see page 42, Part II.).

Figs. 93 and 94 exhibit the fixed light apparatus-the one diagram shewing a central section, and the other a front view. In those figures (which are on the next page) \(f\) is the common focus; \(e\) the refracting belt; \(p p\), the parabolic surfaces; \(n\) mo, the hemisphcrical mirror; and \(s s\), the struts for supporting and stiffening the reflecting plates \(p p\).

Fig. 93.


Fig. 94.


Secondly, There is another combination, which is only applicable to revolving lights, consisting of a hemispherical mirror and a lens, as in the first case, with totally reflecting zones between them, instead of the
paraboloidal surfaces. The zones have the same crosssections as those of Fresnel's small catadioptric apparatus; but, in generating the new zones, the section is supposed to revolve round an horizontal instead of a vertical axis, which, of course, passes through the focus. In this manner the zones lie in parallel planes at right angles to those of the zones in Fresnel's apparatus, which are horizontal. This arrangement is shewn in figures 95 and 96 , in which \(f\) is the common focus of the system ; \(m\), the hemispherical mirror ; and \(z z\), the totally reflecting zones. The distinguishing peculiarity of this arrangement is, that the catadioptric zones, instead of transmitting the light in parallel horizontal plates, as in

Fig. 95.


Fresnel's apparatus, produce, as it were, an extension of the lenticular or quaquaversal action of the central lens, by assembling the light around its axis \(f\) (fig. 96), in the form of concentric hollow cylinders \(z z\).

In so far as photometrical results have been obtained, there is reason to anticipate that an increase of 50 per cent. above the light derived from the common para-
boloidal mirror will result from the holophotal construction.

The last arrangement consists of replacing the hemispherical reflector of metal or silvered glass, shewn at \(m\), in figure 95 , by means of a polyzonal hemisphere, each concentric zone of which has a catadioptric action, like that which is exerted upon rays falling at right angles on the longest side of a right-angled triangular prism. The effect of this arrangement will be best understood by examining fig. 97 , which represents the cross-section of a single concentric zone. The first or inner surface of each zone being concave, and having its centre in the focus F , will, theoretically speaking, receive any ray \(\mathrm{F} e\) as a normal at \(e\), and, no refraction taking place, the ray will pro-
\[
\text { Fig. } 97 .
\]

ceed unaltered in its direction from \(e\) to \(r\); at this point it will be totally reflected in the direction \(r r^{\prime}\), whence it will be a second time totally reflected in the direction \(r^{\prime} e^{\prime}\); at which latter point \(e^{\prime}\), it is also normal to the inner surface, and consequently proceeds unchanged in its path, thus finally returning to the common focus at F. The cross-sections \(C A\) and \(A B\) of the outer surfaces of each zone are, strictly speaking, portions of parabolas facing each other, meeting at the apex \(A\) at
right angles, and also having their common focus in \(\mathbf{F}\), the centre of the hemisphere. In this manner, as already explained, a ray proceeding from the focus falls on the concave or first surface, enters without refraction, is totally reflected at the second surface in a direction tangential to the sphere at the apex of each zone, and passing on, is again reflected at the third surface, and finally emerges from the opposite end of the inner or concave surface without refraction; whence, passing on through the centre of the hemisphere, it becomes a portion of the anterior cone of rays, and being refracted through the lens \(L\), or reflected by the catadioptric rings \(\mathrm{C}, \mathrm{C}, \mathrm{C}, \mathrm{C}\) (see fig. 98 ), finally emerges in the paths shewn by the arrows, and adds its power to the effect of the

Fig. 98.

pencil of rays \(R, R, R\). The union of several of those right-angled zones, arranged in a concentric manner around the horizontal axis of the lens L (fig. 98), completes the hemisphere A, A, A. The central piece, however, will be a species of conoid, having the radius of its base equal to the semichord of its inner surface. Not only would the action of this hemisphere be much more perfect than that of silvered mirrors
or metallic reflectors, by which so much light is absorbed, but it would, moreover, possess the advantage of being less perishable in its polish. The difficulty of grinding the convex faces in a parabolic form might be avoided by using the radius of the circle which osculates the parabolic segment, or by drawing tangents, as in the case of the larger zones, to the curve at \(A\) and at B or C (fig. 97 ) and then tracing the curve from the intersection of the normals. It is obvious that the work will be one of considerable nicety, as the angle of incidence for the rays is confined within very narrow limits; for while, at the apex \(A\), it does not exceed \(45^{\circ}\), it must not, at the points \(B\) and \(C\), be less, when crown-glass is used, than \(41^{\circ} 49\). This would diminish the width of each ring and increase the number required to complete a hemisphere. The adoption of flint-glass would, therefore, be most desirable, as helping to extend the limits within which total reflection might take place, and thus reducing the number of the zones in the hemisphere, and eventually lessening the expense; and a hemisphere about four feet in diameter, composed of seven zones and a central conoìd, might perhaps be found suitable for lights of the first order. I would propose that the junction of the contiguous rings should be formed by means of small copper-dowells or joggles, resting in niches cut in either ring; and the margin of the rings at the points B and C (fig. 97), instead of being accuminated, should be ground in the form of narrow convex and concave bands, similar to those on the rings of the annular lenses, and so inclined as to be frusta of cones meeting in the centre of the hemisphere.

\section*{RUDIMENTARY TREATISE}

ON THE
HISTORY, CONSTRUCTION, and ILLUMINATION
or
LIGHTHOUSES.

BY
ALAN STEVENSON, LL.B., F.R.S.E., M.I.O.E., EIOINEER TO THE BOARD OF NORTHERN LIGHTHOUBEB.

\section*{PART III.}


\author{
LONDON: \\ JOHN WEALE, 59, HIGH HOLBORN. \\ 1850.
}
various general considerations connected WITH LIGHTHOUSES.

In the course of supplying the numerous wants of navigation, it will often be found necessary to cut off, on a given bearing, the beam proceeding from a lighthouse, as a guide to the seaman to avoid some shoal, or as a hint to put about and seek the opposite side of a channel. This is attended with some little practical difficulty, especially in lights from reflectors arranged externally on a circle, because a certain portion of light, chiefly due to the divergence caused by the size of the flames, and partly to the diffraction or inflection of the light, spreads faintly over a narrow sector between the light arc and the dark one. It becomes necessary, of course, to make allowance for this penumbral are by increasing the masked portion of the lantern; and, where a very sharp line of demarcation is required, a board is sometimes placed on the outside of the lightroom, in such a position, and of such length, that while it does not enter the boundaries of the luminous sector, it prevents the more powerful part of the penumbral beam from reaching the observer's eye. This effect is, of course, more conveniently produced, where the circumstances admit of its adoption, by distributing the reflectors round the concave side of the lantern, towards
the land ; but such an arrangement is inapplicable when the illuminated sector exceeds the dark one. I have found, by observation, that the sector intercepted between the azimuth on which the lantern is masked, and that on which total darkness is produced to an observer, at moderate distances, may be estimated at not less than \(3^{\circ}\) for dioptric, and \(7^{\circ}\) for catoptric lights of the highest class.*

Those quantities may therefore serve to guide the lighthouse engineer to approximate more rapidly to his object, as he will generally be safe in increasing the dark sector, by one or other of the above constants according to the kind of apparatus employed. I need not add, that in a matter of this kind, a final appeal to actual observation is, in all cases, indispensable.

A few words on the subject of double lights, naturally spring out of what has been said about the masking of lights. The term double lights is properly and distinctly confined to lights on different levels, but not necessarily (as leading-lights are) in separate towers. The sole object of using double lights is for distinction

\footnotetext{
* The method which I adopted for determining those quantities, was to mask a certain portion of the lantern of a lighthouse subtending an horizontal sector of about \(30^{\circ}\) or \(40^{\circ}\), and at night to fix, by actual observation, at the distance of 5 or 6 miles, two points on the coast between which the light so masked was obscured. The angle included between the lines joining those points and the centre of the lantern was then determined by triangulation next day, and half the difference between the observed angle (which is always the lesser of the two) and the computed subtense of the masked sector of the lantern is, in each case, the amount of the allowance stated in the text.
}
from neighbouring lights ; and they are unquestionably most effective in this respect, when they are placed in the same tower, and when the lower lantern is arranged in the form of a gallery around the outside of the tower, as at Girdleness in Aberdeenshire. In this point of view, therefore, I shall speak of them; and it is obvious that the principal object to be attained is, that the difference of level between them shall be sufficiently great to present the lights, as separate objeets to the eye of the seaman, when at the most distant point whence it is desirable that he should be able to recognise their eharaeteristic appearance. In many cases it is not necessary (but it is certainly always desirable) that the lights should, from the first moment of their being seen, be known as double lights; but in others, it may well consist with safety, that two lights, whieh appear as a single light when first seen at the distance of 20 miles, shall at 15 or 10 miles' distanee be diseovered to be double. Now we should at first be apt hastily to imagine, that all that is required to produce that effect is, to raise the one light above the other to such an extent, that the distance between them shall be somewhat more than a minimum visibile at the most distant point of observation; or, in other words, that the difference of the height of the lights should be such as to subtend to the eye at the point of observation, an angle greater than \(13^{\prime \prime} \cdot 02\), whieh is the subtense of a minimum visibile during the day.* But the effect of irradiation, to whieh I

\footnotetext{
* This quantity is deduced from observations made by my friend Mr James Gardner, while engaged on the Ordnance Survey, and
}
have already alluded, tends to blend together the images of the lights long before their distance apart has become so low a fraction of the observer's distance from the lighthouse, as to subtend so small an angle ; and I have accordingly found by experiments, conducted under various circumstances, and at various distances, that repeated observations gave me \(3^{\prime} 18^{\prime \prime}\) as the mean of the subtenses calculated in reference to the distances at which the lights began to be blended into one.

Adopting this as the smallest angle which the two lights should subtend at the observer's eye, we may find the least vertical distance between them, which will cause them to appear as separate objects, by the following formula :-
\[
\mathrm{H}=2 \Delta \cdot \tan \theta
\]
in which \(\Delta\) is the observer's distance in feet; \(\theta\), half the subtense, \(=1^{\prime} 39^{\prime \prime}\); and H the required height of the tower between the two lights in feet. The following Table gives the height in feet corresponding to the distance in nautic miles, from 1 to 20 inclusive: the heights, which are the bases of similar isosceles triangles, increase, of course, in an arithmetical series :
may be regarded as the extreme limit of visibility, under the most favourable circumstances as to the state of the atmosphere and also the contrast of colours. The observed object, also, was a pole, not a round disc; and it is familiar to every one accustomed to view distant objects, that vertical length is an important constituent in their visibility.
\begin{tabular}{|c|c|c|c|}
\hline \begin{tabular}{c} 
Distance of the \\
observer in \\
Nautic Miles.
\end{tabular} & \begin{tabular}{c} 
Vertical distance \\
in feet between \\
the Lights.
\end{tabular} & \begin{tabular}{c} 
Distance of the \\
observer in \\
Nautic Miles.
\end{tabular} & \begin{tabular}{c} 
Vertical distance \\
in feet between \\
the Lights.
\end{tabular} \\
\hline 1 & 6.02 & 11 & 66.22 \\
2 & 12.04 & 12 & 72.24 \\
3 & 18.06 & 13 & 78.26 \\
4 & 24.08 & 14 & 84.28 \\
5 & 30.10 & 15 & 90.30 \\
6 & 36.12 & 16 & 96.32 \\
7 & 42.14 & 17 & 102.34 \\
8 & 48.16 & 18 & 108.36 \\
9 & 54.18 & 19 & 114.38 \\
10 & 60.20 & 20 & 120.40 \\
\hline
\end{tabular}

Akin to the subject of Double Lights, is that of Leading Lights, the object of which is to indicate to the mariner a given line of direction by their being seen in one line. In most instances, this line of direction is used to point out the central part of a narrow channel ; and the alternate opening of the lights, on either side of their conjunction, serves to indicate to the mariner (who ought to conjoin with his watching of the lights the observation of the elapsed time and also frequent soundings) the proper moment for changing his tack. In some places, the line of conjunction of the lights is placed nearer to one side of a channel than the other, according as the set of the tides, or the position of shoals, may seem to require. In other situations, this line only serves as a cross-bearing to shew the mariner his approach to some
danger, or to indicate his having passed it, and thus to assure him of his entry on wider sea-room. Considerations, similar to those which determine the difference of elevation for double lights, regulate the choice of the distance between two leading lights; but the question is less narrow, and may be generally solved graphically, by simply drawing the lines on an accurate chart of the locality. In some'few situations, the configuration of the coast does not admit of a separation between the lights, sufficient to cause what is called a sharp intersection; but, in most cases, there is room enough to place them so far apart, that but a few yards of deviation in the vessel's course, from the exact line of the conjunction of the lights in one, produces a distinct opening between them on the opposite side of that line. In order to insure the requisite sharpness of intersection, the distance between the lights, wherever attainable, should be not less than one-sixth of the distance between the more seaward of the two towers and that point at which the seaman begins to use the line of conjunction as his guide. I have only to add, that, in situations where the land prevents a considerable separation between leading lights, they should be placed as nearly on one level as is consistent with their being seen as vertically separated, so as, in some measure, to compensate for their horizontal nearness, by rendering their intersection more sharp and striking than it can be where the observer must draw from the upper light an imaginary perpendicular in his mind, and then estimate the separation of the lights by the sine of an angle, which de-
creases as the difference of their apparent elevations increases.

The considerations which enter into the choice of the position and character of the lights on a line of coast, are either, on the one hand, so simple and self-evident as scarcely to admit of being stated in a general form, without becoming mere truisms; or are, on the other hand, so very numerous, and often so complicated, as scarcely to be susceptible of compression into any general laws. I shall not, therefore, do more than very briefly allude to a few of the chief considerations which should guide us in the selection of the sites and characteristic appearance of the lighthouses to be placed on a line of coast. Perhaps those views may be most conveniently stated in the form of distinct propositions:-
1. The most prominent points of a line of coast, or those first made on over-sea voyages, should be first lighted; and the most powerful lights should be adapted to them, so that they may be discovered by the mariner as long as possible before his reaching land.
2. So far as is consistent with a due attention to distinction, revolving lights of some description, which are necessarily more powerful than fixed lights, should be employed at the outposts on a line of coast.
3. Lights of precisely identical character and appearance should not, if possible, occur within a less distance than 100 miles of each other on the same line of coast, which is made by over-sea vessels.
4. In all cases, the distinction of colour should never be adopted except from absolute necessity.
5. Fixed lights, and others of less power, may be more readily adopted in narrow seas, because the range of the lights in such situations is generally less than that of open sea-lights.
6. In narrow seas also, the distance between lights of the same appearance may often be safely reduced within much lower limits than is desirable for the greater sealights. Thus there are many instances in which the distance separating lights of the same character need not exceed 50 miles; and peculiar cases occur in which even a much less separation between similar lights may be sufficient.
7. Lights intended to guard vessels from reefs, shoals, or other dangers, should, in every case where it is practicable, be placed seaward of the danger itself, as it is desirable that seamen be enabled to make the lights with confidence.
8. Views of economy in the first cost of a lighthouse should never be permitted to interfere with placing it in the best possible position ; and, when funds are deficient, it will generally be found that the wise course is to delay the work until a sum shall have been obtained sufficient for the erection of the lighthouse on the best site.
9. The elevation of the lantern above the sca should not, if possible, for sea-lights, exceed 200 feet; and about 150 feet is sufficient, under almost any circumstances, to give the range which is required. Lights placed on high headlands are subject to be frequently wrapped in fog, and are often thereby rendered useless, at times when lights on a lower level might be perfectly
efficient. But this rule must not, and indeed cannot, be strictly followed, especially \({ }^{\circ}\) on the British coast, where there are so many projecting cliffs, which, while they subject the lights placed on them to occasional obscuration by fog, would also entirely and permanently hide from view lights placed on the lower land adjoining them. In such cases, all that can be done is carefully to weigh all the circumstances of the locality, and choose that site for the lighthouse which seems to afford the greatest balance of advantage to navigation. As might be expected, in questions of this kind, the opinions of the most experienced persons are often very conflicting, according to the value which is set on the various elements which enter into the inquiry.
10. The best position for a sea-light ought rarely to be neglected for the sake of the more immediate benefit of some neighbouring port, however important or influential; and the interests of navigation, as well as the true welfare of the port itself, will generally be much better served by placing the sealight where it ought to be, and adding, on a smaller scale, such subsidiary lights as the channel leading to the entrance of the port may require.
11. It may be held as a general maxim, that the fewer lights that can be employed in the illumination of a coast the better, not only on the score of economy, but also of real efficiency. Every light needlessly erected may, in certain circumstances, become a source of confusion to the mariner ; and, in the event of another light being required in the neighbourhood, it becomes a deduction from the means of distinguishing it from the lights which existed previous to its establishment. By
the needless erection of a new lighthouse, therefore, we not only expend public treasure, but waste the means of distinction among the neighbouring lights.
12. Distinctions of lights, founded upon the minute estimation of intervals of time between flashes, and especially on the measurement of the duration of light and dark periods, are less satisfactory to the great majority of coasting seamen, and are more liable to derangement by atmospheric changes, than those distinctions which are founded on what may more properly be called the characteristic appearance of the lights, in which the times for the recurrence of certain appearances differ so widely from each other as not to require for their detection any very minute observation in a stormy night. Thus, for example, flashing lights of five seconds' interval, and revolving lights of half a minute, one minute, and two minutes, are much more characteristic than those which are distinguished from each other by intervals varying according to a slower series of \(5^{\prime \prime}, 10^{\prime \prime}\), \(20^{\prime \prime}, 40^{\prime \prime}\), \&c.
13. Harbour and local lights, which have a circumscribed range, should generally be fixed instead of revolving; and may often, for the same reason, be safely distinguished by coloured media. In many cases also, where they are to serve as guides into a narrow channel, the leading lights which are used should, at the same time, be so arranged as to serve for a distinction from any neighbouring lights.
14. Floating lights, which are very expensive and more or less uncertain, from their liability to drift from their moorings, as wcll as defective in power, should never be employed to indicate a turning-point in a navi-
gation in any situation where the conjunction of lights on the shore can be applied at any reasonable expense.

The spheroidal form of the earth requires that the height of a lighthouse tower should increase proportionally to the difference between the earth's radius and the secant of the angle intercepted between the normal to the spheroild at the lighthouse and the normal at the point of the light's occultation from the view of a distant observer. The effect of atmospheric refraction, however, is too considerable to be neglected in estimating the range of a light, or in computing the height of a tower which is required to give to any light a given range ; and we must, therefore, in accordance with the influence of this element, on the one hand increase the range due to any given height, and vice versa reduce the height required for any given range, which a simple consideration of the form of the globe would assign. In ascertaining this height, we may proceed as follows:Referring to the accompanying figure (No. 99), in Fig. 99.

which \(\mathrm{S}^{\prime} d \mathrm{~L}^{\prime}\) is a segment of the ocean's surface, O the centre of the earth, L'L a lighthouse, and \(S\) the position of the mariner's eye, we obtain the value of \(\mathrm{LL}^{\prime}=\mathrm{H}^{\prime}\), the height of the tower in feet by the formula,
\[
\begin{equation*}
\mathrm{H}^{\prime}=\frac{2 l^{2}}{3} \tag{1.}
\end{equation*}
\]
in which \(l=\) the distance in English miles L' \(d\) at which the light would strike the ocean's surface. We then reduce this value of \(\mathrm{H}^{\prime}\) by the correction for mean refraction, which permits the light to be seen at a greater distance, and which \(=\frac{2 l^{2}}{21}\),

So as to get, \(\quad \mathrm{H}=\frac{2 l^{2}}{3}-\frac{2 l^{2}}{21}=\frac{4 l^{2}}{7}\)
an expression which at once gives the height of the tower required, if the eye of the mariner were just on the surface of the water at \(d\), where the tangent between his eye at \(S\) and the light at \(L\) would touch the sea. We must, therefore, in the first instance, find the distance \(d \mathrm{~S}=l^{\prime}\), which is the radius of the visible horizon due to the height \(\mathrm{SS}^{\prime}=h\) of his eye above the water, and is, of course, at once obtained conversely by the expression :-
\[
\begin{equation*}
l^{\prime}=\frac{\sqrt{7 h}}{2} \tag{4.}
\end{equation*}
\]

Deducting this distance from SL, the whole effective range of the light, we have \(\mathrm{L} d=l\), and operating with this value in the former equation,
\[
\mathrm{H}=\frac{4 l^{2}}{7}
\]
we find the height of the tower which answers the conditions of the case.* From the above data the following Table has been computed.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline  &  & \[
\begin{gathered}
\lambda^{\prime} \\
\text { Lengths } \\
\text { in } \\
\text { Nautical } \\
\text { Miles. }
\end{gathered}
\] & \begin{tabular}{l}
H. \\
Heights in Feet
\end{tabular} & \[
\begin{gathered}
\lambda \\
\text { Lengths } \\
\text { in } \\
\text { English } \\
\text { Miles. }
\end{gathered}
\] & \[
\begin{gathered}
\lambda^{\prime} \\
\text { Lengths } \\
\text { in } \\
\text { Nautica! } \\
\text { Miles. }
\end{gathered}
\] &  & \begin{tabular}{l}
\(\lambda\) \\
Lengths in \\
English Miles.
\end{tabular} & \[
\begin{gathered}
\lambda^{\prime} \\
\text { Lengths } \\
\text { in } \\
\text { Nautical } \\
\text { Miles. }
\end{gathered}
\] \\
\hline 5 & 2.958 & 2.565 & 70 & 11.067 & 9-598 & 250 & 20.916 & \(18 \cdot 14\) \\
\hline 10 & \(4 \cdot 184\) & \(3 \cdot 628\) & 75 & 11.456 & 9.935 & 300 & 22.912 & 19.87 \\
\hline 15 & \(5 \cdot 123\) & \(4 \cdot 443\) & 80 & 11•832 & \(10 \cdot 26\) & 350 & 24.748 & 21.46 \\
\hline 20 & 5.916 & 5•130 & 85 & 12196 & \(10 \cdot 57\) & 400 & 26.457 & \(22 \cdot 94\) \\
\hline 25 & 6.614 & \(5 \cdot 736\) & 90 & 12.549 & 10.88 & 450 & 28.062 & \(24 \cdot 33\) \\
\hline 30 & 7.245 & 6.283 & 95 & 12.893 & \(11 \cdot 18\) & 500 & 29.580 & 25.65 \\
\hline 35 & 7.826 & 6.787 & 100 & 13.228 & \(11 \cdot 47\) & 550 & 31.024 & 26.90 \\
\hline 40 & \(8 \cdot 366\) & 7.255 & 110 & \(13 \cdot 874\) & 12.03 & 600 & 32.403 & \(28 \cdot 10\) \\
\hline 45 & 8.874 & \(7 \cdot 696\) & 120 & 14.490 & 12.56 & 650 & 33.726 & \(29 \cdot 25\) \\
\hline 50 & \(9 \cdot 354\) & 8.112 & 130 & 15.083 & 13.08 & 700 & \(35 \cdot 000\) & \(30 \cdot 28\) \\
\hline 55 & 9.811 & 8.509 & 140 & 15.652 & 13.57 & 800 & \(37 \cdot 416\) & \(32 \cdot 45\) \\
\hline 60 & \(10 \cdot 246\) & 8.886 & 150 & \(17 \cdot 201\) & 14.91 & 900 & \(39 \cdot 836\) & \(34 \cdot 54\) \\
\hline 65 & 10.665 & \(9 \cdot 249\) & 200 & 18.708 & 16.22 & 1000 & 41-833 & 36.28 \\
\hline
\end{tabular}

If the distance at which a light of given height can

\footnotetext{
* In the above expressions \(l\) and \(l^{\prime}\) are given in English miles, which in Scotland may be considered as bearing to nautical miles the ratio of 5280 to 6088 . In order to convert a distance given in nautical miles to English miles, all that is needful is to add the \(\log\) of the number of nautical miles to \(\log 5280\), and subtract \(\log 6088\).
}
be seen by a person on a given level be required, it is only needful to add together the two numbers in the column of lengths \(\lambda\) or \(\lambda^{\prime}\) (according as Nautical or English miles may be sought), corresponding to those in the column of heights H , which represent respectively the height of the observer's eye and the height of the lantern above the sea. When the height required to render a light visible at a given distance is required, we must seek first for the number in \(\lambda\) or \(\lambda^{\prime}\) corresponding to the height of the observer's eye, and deduct this from the whole proposed range of the light, and opposite the remainder in \(\lambda\) or \(\lambda^{\prime}\) seek for the corresponding number in H .

A considerable practical defect in all the lighthouse lanterns which I have ever seen, with the exception of those recently constructed for the Scotch lighthouses, consists in the vertical direction of the astragals, which, of course, tend to intercept the whole or a great part of the light in the azimuth which they subtend.* The consideration of the improvement which I had effected in giving a diagonal direction to the joints of the fixed refractors (see page 46, Part II.), first led me to adopt a diagonal arrangement of the framework which carries the cupola of zones, and afterwards for the astragals of the lantern. Not only is this direction of the astragals more advantageous for equalizing the effect of the light; but the greater stiffness and strength which such an arrangement gives to the framework of the lantern,

\footnotetext{
* I must also except the small pier light at Kirkcaldy, erected (I believe in 1836) by my friend Mr Edward Sang.
}
make it safe to use more slender bars, and thus also absolutely less light is intercepted. The panes of glass at the same time become triangular, and are necessarily stronger than rectangular panes of equal surface. This form of lantern is extremely light and elegant, and is shewn, with detailed drawings of some of its principal parts, in Plate X. To avoid the necessity of painting, which, in situations so exposed as those which lighthouses generally occupy, is attended with many inconveniences and no small risk, the framework of the lantern is now formed of gun-metal and the dome is of copper. A lantern for a light of the first order, 12 feet in diameter, and with glass frames 10 feet high, costs, when glazed, about \(£ 1260\). In order to give the lightkeepers free access to cleanse and wash the upper panes of the lantern (an operation which in snowy weather must sometimes be frequently repeated during the night), a narrow gangway, on which they may safely stand, is placed on the level of the top of the lower panes, and at the top of the second panes, rings are provided of which the lightkeepers may lay hold for security in stormy weather. A light trap-ladder is also attached to the outside of the lantern by means of which there is an easy access to the ventilator on the dome.

Great care is bestowed on the glazing of the lantern, in order that it may be quite impervious to water, even during the heaviest gales. When iron is used for the frames, they are carefully and frequently painted; but gun-metal, as just noticed, is now generally used in the Scotch lighthouses. There is great risk of the glass
plates being broken by the shaking of the lantern during high winds; and as much as possible to prevent this, various precautions are adopted. The arris of each plate is always carefully rounded by grinding; and grooves about \(\frac{1}{2}\) inch wide, capable of holding a good thickness of putty, are provided in the astragals for receiving the glass, which is \(\frac{\lambda}{~}\) inch thick. Small pieces of lead or wood are inserted between the frames and the plates of glass against which they may press, and by which they are completely separated from the more unyielding material of which the lantern-frames are composed. Panes glazed in frames padded with cushions, and capable of being temporarily fixed in a few minutes, in the room of a broken plate, are kept ready for use in the store-room. Those framed plates are called stormpanes, and have been found very useful on several occasions, when the glass has been shattered by large seabirds coming against it in a stormy night, or by small stones violently driven against the lantern by the force of the wind.

The ventilation of the lanterns forms a most important element in the preservation of a good and efficient light. An ill-ventilated lantern has its sides continually covered with the water of condensation, which is produced by the contact of the ascending current of heated air; and the glass, thus obscured, obstructs the passage of the rays and diminishes the power of the light. In the Northern Lighthouses, ventilators, capable of being opened and shut at pleasure, so as to admit from without a supply of air when required, are provided in the
parapet-wall on which the lantern stands; the lantern roof also is surmounted by a cover which, while it closes the top of an open cylindric tube against the entrance of rain, and descends over it only so far as is needful for that purpose, still leaves an open air-space between it and the dome. This arrangement permits the current of heated air, which is continually flowing from the lantern through the cylindric tube, to pass between it and the outer cover, from which it finally escapes to the open air through the space between the cover and the dome. The door which communicates from the lightroom through the parapet to the balcony outside, is also made the means of ventilating the lightroom ; and, for that purpose, it is provided with a sliding bolt at the bottom, which, being dropped into one or other of the holes cut in the balcony for its reception, serves to keep the door open at any angle that may be found necessary. A useful precaution was introduced by my predecessor, as Engineer to the Northern Lights Loard, in order to prevent the too rapid condensation of heated air on the large internal surface of the lantern roof, , which consists in having two domes with an air-space between them, as shewn in the enlarged diagrams in Plate X.

An important improvement in the ventilation of lighthouses was, some years ago, introduced by Dr Faraday into several of the lighthouses belonging to the Trinity House, and has since been adopted in all the dioptric lights belonging to the Commissioners of Northern Lighthouses. After mentioning several proofs of ex-
tremely bad ventilation in lighthouses, Dr Faraday thus describes his apparatus:*
" The ventilating pipe or chimney is a copper tube, 4 inches in diameter, not, however, in one length, but divided into three or four pieces; the lower end of each of these pieces for about \(1 \frac{1}{2}\) inch is opened out into a conical form, about \(5 \frac{1}{2}\) inches in dłameter at the lowest part. When the chimney is put together, the upper end of the bottom piece is inserted about \(\frac{1}{2}\) inch into the cone of the next piece above, and fixed there by three ties or pins, so that the two pieces are firmly held together; but there is still plenty of air-way or entrance into the chimney between them. The same arrangement holds good with each succeeding piece. When the ventilating chimney is fixed in its place, it is adjusted so that the lamp-chimney enters about \(\frac{1}{2}\) inch into the lower cone, and the top of the ventilating chimney enters into the cowl or head of the lantern.
" With this arrangement, it is found that the action of the ventilating flue is to carry up every portion of the products of combustion into the cowl; none passes by the cone apertures out of the flue into the air of the lantern, but a portion of the air passes from the lantern by these apertures into the flue, and so the lantern itself is in some degree ventilated.
"The important use of these cone apertures is, that when a sudden gust or eddy of wind strikes into the cowl of the lantern, it should not have any effect in disturbing or altering the flame. It is found that the wind

\footnotetext{
* Minutes of Institution of Civil Engineers, vol. i., p. 207.
}
may blow suddenly in at the cowl, and the effect never reaches the lamp. The upper, or the second, or the third, or even the fourth portion of the ventilating flue might be entirely closed, yet without altering the flame. The cone junctions in no way interfere with the tube in carrying up all the products of combustion; but if any downward current occurs, they dispose of the whole of it into the room without ever affecting the lamp. The ventilating flue is in fact a tube, which, as regards the lamp, can carry everything up but conveys nothing down."

The advantages of this arrangement, as applied to the Northern Lighthouses, were much less palpable than those which are described in the beginning of Dr Faraday's paper, because their ventilation was very good before its introduction; and the flame in particular was perfectly steady, being by no means subject to derangement from sudden gusts of wind from the roof in the manner noticed above.

All the lighthouses in the district of the Scotch Commissioners are under the charge of at least two lightkeepers, whose duties are to cleanse and prepare the apparatus for the night illumination, to mount guard singly after the light is exhibited, and to relieve each other at stated hours, fixed by the printed regulations and instructions, under which they act. The rule is, that no keeper on watch shall, under any circumstances, leave the lightroom until relieved by his comrade; and, for the purpose of cutting off all pretext for the neglect of this universal law, the dwelling-houses are built close to the Light Tower, and means are provided for
making signals directly from the lightroom to the sleeping apartments below. The signals are communicated by air-tubes (Plate XII.), which pass from the lightroom to the sleeping-apartments in the houses, and through which, by means of a small piston, or a puff of wind from the mouth, calls can be exchanged between the keepers. The man on guard in the lightroom, at the end of the watch, or on any sudden emergence, may thus summon his comrade from below, who, on being thus called, answers by a counter-blast, to shew that the summons has been heard and will be obeyed. For the purpose of greater security, in such situations as the Bell Rock and the Skerryvore, four keepers are provided for one lightroom; one being always ashore on leave with his family, and the other three being at the lighthouse; so that, in case of the illness of one lightkeeper, an efficient establishment of two keepers for watching the light may remain. At all the land-lighthouses also, an agreement is made with some steady person residing in the neighbourhood, who is instructed in the management of the light and cleansing of the apparatus, and comes under an obligation to be ready to do duty in the light-room when called upon, in the event of the sickness or absence of one of the lightkeepers. This person is called the occasional keeper, and receives pay only while actually employed at the lighthouse; but in order to keep him in the practice of the duty, he is required to serve in the lightroom for a fortnight annually in the month of January. For the more minute details of the lightkeeper's duty, I would refer the reader to the Instructions already alluded to, which will be found at the end of this volume.

Each of the two lightkeepers has a house for himself and family, both being under a common roof, but entering by separate doors, as shewn in Plates XI. and XII., which exhibit the buildings for the new lighthouse at Ardnamurchan Point, on the coast of Argyllshire. The principal keeper's house consists of six rooms, two of which are at the disposal of the visiting officers of the Board whose duty in inspecting the lighthouse or superintending repairs, may call them to the station; and the assistant has four rooms, one of which is used as a barrack-room for the workmen, who, under the direction of the foreman of the lightroom works, execute the annual repairs of the apparatus.

The early lighthouses contained accommodation for the lightkeepers in the tower itself; but the dust caused by the cleaning of those rooms in the tower was found to be very injurious to the delicate apparatus and machinery in the lightroom. Unless, therefore, in situations such as the Eddystone, the Bell Rock, or the Skerryvore, where it is unavoidable, the dwellings of the lightkeepers ought not to be placed in the Light Tower, but in an adjoining building.

Great care should be bestowed to produce the utmost cleanliness in everything connected with a lighthouse, the optical apparatus of which is of such a nature as to suffer materially from the effect of dust in injuring its polish. For this purpose covered ash-pits are provided at all the dwelling-houses, in order that the dust of the fire-places may not be carried by the wind to the lightroom; and, for similar reasons, iron floors are used for the lightrooms instead of stone, which is often liable to
abrasion, and all the stonework near the lantern is regularly painted in oil.

If, in all that belongs to a lighthouse, the greatest cleanliness be desirable, it is in a still higher degree necessary in every part of the lightroom apparatus, without which the optical instruments and the machinery will neither last long nor work well. Every part of the apparatus, whether lenses or reflectors, should be carefully freed from dust before being either washed or burnished; and without such a precaution, the cleansing process would only serve to scratch them. For burnishing the reflectors, prepared rouge (tritoxide of iron) of the finest description, which should be in the state of an impalpable powder of a deep orange-red colour, is applied, by means of soft chamois skins, as occasion may require ; but the great art of keeping reflectors clean consists in the daily, patient, and skilful application of manual labour in rubbing the surface of the instrument with a perfectly dry, soft, and clean skin, without rouge. The form of the hollow paraboloid is such, that some practice is necessary in order to acquire a free movement of the hand in rubbing reflectors; and its attainment forms one of the principal lessons in the course of the preliminary instruction, to which candidates for the situation of a lightkeeper are subjected at the Bell Rock Lighthouse. For cleansing the lenses and glass mirrors, spirit of wine is used. Having washed the surface of the instrument with a linen cloth steeped in spirit of wine, it is carefully dried with a soft and dry linen rubber, and finally rubbed with a fine chamois skin, free from any dust which would injure the polish of the glass,
as well as from grease. It is sometimes necessary to use a little fine rouge with a chamois skin, for restoring any deficiency of polish which may occur from time to time; but in a well-managed lighthouse this application will seldom, if ever, be required.

The machinery of all kinds, whether that of the mechanical lamp or the revolving apparatus, should also be kept scrupulously clean, and all the journals should be regularly and carefully oiled.

As I have had frequent occasion to speak of the comparative power of lights, it will not be out of place to present the reader with a few practical observations, chiefly drawn from the excellent work of M. Peclet to which I have so often referred, on the measurement of the intensity of lights by the method of shadows.

The intensity of light decreases as the observer recedes from the luminous body, in proportion to the square of his distance. Suppose a beam of light to procced from a radiant at F (fig. 100), and we shall have the rays which,

Fig. 100.

of course, move in straight lines, gradually receding from each other, as \(b, b^{\prime}, b^{\prime \prime}, b^{\prime \prime \prime}\), and \(c, c^{\prime}, c^{\prime \prime}, c^{\prime \prime \prime}\), so that the section of the beam will increase with the distance \(\mathbf{F} b\), and \(\mathbf{F} c\); and the same number of rays, being thus
spread over spaces continually increasing, will illuminate the surfaces with a less intensity. This decrease of intensity will, therefore, be in the inverse ratio of the extent of the transverse parallel sections of the luminous cones at \(b\) and \(c\), which, we know, increase as the square of their distances from the apex of the cone at F. Hence we conclude, that the intensity of any section of a divergent beam of light decreases as the square of its distance from the radiant. This law furnishes us with a simple measure of the comparative intensity of lights. If we suppose two lights so placed that they may separately illuminate adjacent portions of a vertical screen of paper, we may, by repeatedly comparing the luminousness of those surfaces, and moving one of the lights farther from, or nearer to the screen, at length cause the separate portions of the paper to become equally luminous. This arrangement, however, has many practical difficulties, which I shall not wait to specify; but shall at once indicate a more simple and equally correct mode of obtaining the same result, by means of the shadows cast by the lights from an opaque rod, in a vertical position at 0 (fig. 101), placed between them, and a screen Fig. 101.

covered with white paper on which the shadows fall. It is obvious that the light at F would cause the object \(O\) to cast a shadow at SS, while the light at \(\mathrm{F}^{\prime}\) would cast a shadow at \(\mathrm{S}^{\prime} \mathrm{S}^{\prime}\). But while the shadow at S would still receive light from \(F^{\prime}, S^{\prime}\) would receive light from F, so that those two shadows are, in fact, the only portions of the screen which are each illuminated only by one of the lights, while every other portion of its surface receives light from both the radiants at F and \(\mathrm{F}^{\prime}\). If we suppose \(\mathbf{F}\) to be the weaker light, we can bring it nearer the screen, until the shadow \(\mathrm{S}^{\prime} \mathrm{S}^{\prime}\), shall become similar in appearance to the shadow S S; and we shall have the ratio of the intensity of the light at F to that of the light at \(\mathrm{F}^{\prime}\), as \(\left(\mathrm{F} \mathrm{S}^{\prime}\right)^{2}\) is to \(\left(\mathrm{F}^{\prime} \mathrm{S}\right)^{2}\), which distances must be measured with the greatest exactness. Such is the mode commonly used in estimating the comparative intensities of two lights; but there are various precautions which are needful in order to prevent errors in comparing the deepness of the shadows, and to insure the greatest attainable accuracy in the estimate of the power of the lights, which I shall endeavour briefly to describe.
The difficulties of estimating the deepness or sharpness of the shadow is very great, and many persons seem quite incapable of arriving at any right judgment in this matter. The same person also will discover such unaccountable variations in his decision after observations made at short intervals of time, as, one would think, can only arise from a sudden change of the intensity of one or both lights. M. Peclet, in his Traité de l'éclairage, gives, as the result of his experience (and I can
fully confirm his result by my own), that those differences depend less frequently on any real difficulty of estimating the deepness of the shadows, than on variations in the position of the observer, or rather in the angle at which he views the shadows, and that, consequently, in proportion to the distance between the two shadows, this source of error is increased. Any thing like a glossy texture of the surface of the screen, which then, of course, becomes a reflector, also tends to aggravate this evil. Thus, if the two lights which are to be compared be placed on a table, in such situations as to spread pretty far apart on the screen the shadows of a vertical rod placed between them; and if the shadow nearer to the observer seem to be a little deeper or sharper than the other, let the observer look at them from the other side of the table, and their difference will be reversed, and that which seemed the paler, will become the deeper. Again, if the difference between the two shadows be very great when seen from the right side of the screen, it may happen that, on viewing them from the left hand, the difference may still be in favour of the same shadow, but in a much less degree.
" When I observed this effect," says M. Peclet,* "I tried to view the shadows through a transparent screen, but I remarked the same variations. They were indeed even more sensible; for a variation in the distance of the eye of a few centimètres, made a prodigious change in the deepness of the shadows. I observed also that

\footnotetext{
* Traité de l'Eclairage, p. 214.
}
the shadow was much deeper when seen in the line of the light, and that, in every other direction, it became paler in proportion as the eye receded from that direction.
" In all the cases which I have just described, the differences of the tints when the position is changed, increase in proportion as the shadows are farther scparate; and they grow very minute when the shadows are almost touching each other.
" Let A B (fig. 102) be a white opaque surface, \(a\), a
Fig. 102.

luminous body, and \(m\), a black opaque body, then the shadow \(b^{\prime}\) cast on A B, will appear deeper when observed from \(P\), than as seen from \(Q\). This is a fact which may be easily verified, and the cause of which is easily conceived. In fact, the surface A B, although it disperses the light, must still reflect more of it, in the directions in which the regular reflection takes place; and hence the rays which are reflected round about the shadow, must have a greater intensity in the direction of \(\mathbf{P}\) than in that of \(Q\), and, consequently, the shadow \(b^{\prime}\) must appear deeper from the point \(\mathbf{P}\) than from \(Q\).
"If we now place (fig. 103) two lights in front of the
Fig. 103.

screen AB , at such distances that the two shadows \(a^{\prime}\) and \(b^{\prime}\) should have equal intensities, it is evident that if the eye be placed at \(\mathbf{P}\), the shadow \(b^{\prime}\) must appear more intense than the shadow \(a^{\prime}\), and that the reverse will take place if the eye be at \(Q\). But the difference which is then observed, arises not only from the difference in the brightness of the parts surrounding the shadows, but also from a difference in the intensity of the shadows themselves; for the shadow \(b^{\prime}\) is illuminated by \(b\), and radiates much more towards \(Q\) than towards P ; and, on the contrary, the shadow \(a^{\prime}\), which is illuminated by \(a\), radiates much more towards P than towards Q . We perceive also why the differences of the tints increase with the separation of the two shadows, and why they become very small when the shadows touch each other; it is because, in proportion as the shadows are farther apart, each of them is illuminated more obliquely, and a greater quantity of light is radiated (by reflection) in the regular direction. When they touch each other, on the contrary, they are illuminated almost perpendicularly, and consequently the shadows radiate light almost equally on either side.
" Those anomalies of a like kind which are observed when the shadows are viewed through a translucent body, such as paper or linen, may be referred to a similar cause. We know, in fact, that, in looking through a translucent medium, we always, more or less, distinctly perceive the luminous body behind it, and, also, that there is a very large proportion of the rays which traverse the body, which stray but a little from the direction which they would follow if the substance were absolutely transparent. Consequently, the space which surrounds the shadow is more luminous in proportion as we come nearer to the direction of the shadow ; and as the absolute intensity of the shadows diminishes as we come nearer to the direction of the rays which light them, those two effects concur to increase the intensity of that shadow to which the eye is nearer.
"As the dispersion by reflection is much more complete than by refraction, the variations of which we have just spoken are much greater with a transparent screen, through which the shadows are viewed, than with an opaque screen (from which they are reflected)."

The mode of comparing lights adopted by M. Peclet is thus described by himself. "I view," says he, " first, the two shadows in such a manner that both of them may be seen in succéssion from either side of the body which produces them, and at equal distances. For this purpose I use a good opera-glass. I alter the distance of the flames until in those two positions I perceive the differences (of the intensity in the shadows) to be in opposite directions. The distances of the lamps may then be considered as very nearly in the proper propor-
tion for producing equal shadows, and to make them exactly so, the differences, which are observed on either side (of the centre line between them), should be equal; and, of course, the two shadows themselves, seen at one moment from either side of the opaque body, should be perfectly equal also.* These three observations, which mutually serve to verify or correct each other, will lead, with a little practice, to very great precision in the result. We may, also, by using a narrow screen, bring the shadows sufficiently near to touch each other; the variations of the tints then become very small by any change of our position, and we may, in this case, rest content with observing them from one point. To get rid of large penumbrae, which are always an obstacle in forming a right estimate of the tints of the shadows, I place the opaque body very near the screen.
"When we wish to make a great many observations, it is very convenient to mark divisions on the table (which carries the lights), in order to read off, by means of them, the distance of the lamps from the shadows which they illuminate. By this means, each observation need not occupy more than two minutes. I generally use a table CC DD (fig. 104), about two mètres long ( 6 feet 6 inches), by 80 centimètres wide ( 2 feet 8 inches). At one end I place the screen AB , covered

\footnotetext{
* I prefer to view the exterior portions of both shadows from the central line itself, in which case the opaque rod stands between them, because, in this manner, I obtain a more correct comparison by the direct contrast of the surfaces than by successive views of them, however quickly taken.
}
A. S.
with white paper, dull (or not glazed), and kept in a
Fig. 104.

vertical plane by two small pieces \(P\) and Q . Through the point M, the centre of the opaque body, I draw two lines \(\mathrm{M} f\) and \(\mathrm{M} g\), equally inclined to the central line \(x y\), whose extremities \(b^{\prime}, a^{\prime}\) are the axes of the two shadows. These lines must be inclined in such a manner that the distance of the shadows may be a little less than the diameter of the opaque body, or so that they may actually touch each other, according to the mode of observing which you wish to follow. These lines \(\mathbf{M} f\), \(\mathrm{M} g \mathrm{I}\) divide into decimètres and centimètres (starting from the points \(a^{\prime}, b^{\prime}\) ), and over those lines I place the centres of the flames, so that the distance between the shadows remains always the same, whatever may be the distance of the lamps. In order to determine the distance of each lamp from the shadow which it illuminates, we ought, strictly speaking, to take the distance of the centre of the flame \(b\) from the point \(a^{\prime}\); but as the distance from the point \(b\) to the point \(a^{\prime}\) differs little from the distance between the points \(b\) and \(b^{\prime}\), we may assume the
latter for the former, without causing any sensible error. That distance may be very conveniently ascertained by taking the half of the sum of the distances of the two extremities \(z\) and \(z^{\prime}\) of the diameter of the pedestal of the lamp. When the burner is not placed over the centre of the pedestal, we may suspend from it a small plummet, whose point will touch some division and indicate the distance between the centre of the burner and the shadow.
" When the lights are coloured, the shadows are coloured also, and it is then far more difficult to judge accurately of their intensity. They may, in that case, be much better seen from the point \(x\), as the black opaque body which is interposed between them renders the difference of colour less sensible to the eye.
"The opaque body M is a cylindric rod of iron, whose upper part is blackened in the flame of a lamp, in order to prevent the reflection which might interfere with the sharpness (netteté) of the shadows, and to make them more distinct when they are viewed from the point \(x\)."*

I shall make a few trifling additions to M. Peclet's clear description of his excellent mode of measuring the intensity of lights. It is, of course, presumed throughout that the centres of the flames should be on one level; and I have found it most convenient to place the lamps on small carriages with rollers, which are guided by means of fine strips of wood nailed along the table in the directions \(g \mathrm{M}\) and \(f \mathrm{M}\), and carrying the divided

\footnotetext{
* Those who feel a curiosity to look farther into this subject may consult Count Rumford's elaborate paper in the Phil. Trans. for 1794, p. 67.
}
scales of centimètres. This affords the means of making any slight change in the position of the lamps so easily, as entirely to avoid the disturbance of the flame which ensues from lifting the lamp and readjusting it in another position; and will, in practice, be found very convenient when many observations are to be made. I have already said that my own experience has satisfied me that, with the aid of a good opera-glass, the central observation of the two shadows, with the opaque rod between them, is by far the best, and conducts, at once, to a result which is confirmed by the observations of two assistants who watch the shadows at the same time on opposite sides of the table, and at equal distances from them. I have found it convenient in comparing lights, to cover the table with dull black linen cloth, and to surround it with curtains of the same material, hung from slender brackets, in such a manner as to leave space for the observer to move freely round the table within them. The curtains prevent reflection from the walls of the chamber in which the experiments may be conducted, and also lessen the disturbing effects of currents of air. When a comparison of the intensity, and not of the aggregate power of two flames, is to be made, it is necessary to adopt the precaution of inclosing the lights in opaque boxes, with slits of equal area in each, placed on the same level, and so arranged, in reference to the flames, as to be directly opposite the brightest portion of each. After what has been said, it will be almost needless to add that the quotient of the square of the greater observed distance divided by the lesser, is the ratio of the illuminating power of the two flames.

The most convenient mode of registering observations, and that which is generally practised, is in the form of a Table like the following :-
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{Trials.} & \multicolumn{2}{|r|}{Distance.} & \multicolumn{2}{|l|}{Squares of Distance.} & \multicolumn{2}{|l|}{Illuminating Power, or Quotient of Squares.} \\
\hline & Lamp A. & Lamp B. & Lamp A. & Lamp B. & Lamp A. & Lamp B. \\
\hline 1 & 143 & 140 & 20,449 & 19,700 & \(1 \cdot 00\) & 0.958 \\
\hline 2 & 117 & 114 & 13,689 & 12,996 & \(1 \cdot 00\) & 0.949 \\
\hline
\end{tabular}

As a standard lamp by which to test others, I believe none will be found superior to the best Carcel lamp, which has a clockwork movement, and whose flame continues to increase in power for about four hours after it is lighted; after which it maintains its state permanently, until the supply of oil fails. This fact was verified by M. Peclet with the greatest care. "I took," says he, "two similar lamps. They were lighted at the same time, and their relative intensities were measured. One was then extinguished, without touching the wick, and its clockwork movement was stopped. One hour afterwards, I set the clockwork in motion and relighted the lamp, but without touching the wick. It was found in the same state as at the first comparison, and I measured its intensity in reference to the first. Those experiments I repeated every hour, and these are the results which I obtained. The lamp which I call No. 1, is that which remained continually burning; No. 2 is that which
was only lighted during the continuance of the (successive) observations."
\begin{tabular}{|c|c|c|}
\hline \multirow{2}{*}{\begin{tabular}{c} 
Times of \\
Observation.
\end{tabular}} & \multicolumn{2}{|c|}{\begin{tabular}{c} 
Intensities. \\
\cline { 3 - 4 } \\
Lamp, No. 1.
\end{tabular}} \\
\hline Lamp, No. 2. \\
\hline 5 & 30 & 100 \\
6 & 30 & 103 \\
7 & 30 & 106 \\
8 & 30 & 110 \\
9 & 30 & 117 \\
10 & 30 & 117 \\
11 & 30 & 117 \\
12 & 30 & 117 \\
\hline
\end{tabular}

This curious scale of increase in power, seems to be solely due to a peculiarity of the manner in which the lamp, that derives its supply of oil by clockwork, becomes heated; and the effect may be described as follows: The heating of the wicks, the chimney, and the oil in this burner, as in that of all other lamps, tends to increase the light; but, in an ordinary lamp, acting by a constant pressure, this maximum of heat is soon attained; whereas in the clockwork-lamp, into the burner of which the oil is thrown up by a pump, the whole of the oil in the cistern must reach its maximum temperature before the best effect of that lamp is produced. After this state has been reached, there is no disturbing influence at work, and the lamp burns steadily as long as the oil lasts.

I have myself tried what may naturally appear to be the most simple mode of obtaining an unvarying stan-dard-light, by employing a gas-burner, supplied from a gasometer under a constant pressure; but I found it very difficult to obtain satisfactory proof of the constancy of the pressure; and in a large town, where there are many burners around one, their lighting or extinction is found to exercise a material influence in changing the condition of the flame. I must confess that I have always been disappointed in attempting to use a gas-flame as a standard of comparison.

A very convenient form of photometer has been invented by Professor Bunsen of Marburg. It consists of a disc of writing paper, in the centre of which is painted, with a camel-hair pencil dipped in spermaceti dissolved in naphtha, a ring, surrounding a small circular spot. At a fixed distance behind this dise is placed a light which is not liable to variations in intensity; and one of the two lights which are to be compared is placed in front of the paper disc, and is made to approach it, until the spot within the spermaceti ring disappears. The distance of the light is measured, and the light is then removed; after which the other light is applied in the same manner, and the distance at which the spot disappears is measured also. The ratio of the squares of the two distances from the paper disc will accurately represent the relative brightness of the two lights thus compared. The disappearance of the spot, within the ring, is obviously caused by the sum of the lights reflected from it, and transmitted through it, becoming equal to the sum of the lights reffected from
and transmitted by the translucent ring of spermaceti. It is of course assumed, that the light behind the paper disc remains constant in intensity and distance; and, consequently, the light transmitted by the central spot, and also that transmitted by the ring will be invariable also. Since, then, each of the lights, which are compared successively, cause the central spot to disappear, it is evident that in both cases the light which falls from them on the paper must, at its surface, be equally bright; and, therefore, the ratio of the squares of their distances from the paper is a true measure of their relative brightness.

A less accurate approximation may also be made, by substituting for the paper disc of Bunsen, one of the small revolving beads of silvered glass, by means of which Professor Wheatstone describes his beautiful luminous curves. When a single bead is made to revolve in a ellipse, so prolate as to appear like a straight line, and having its conjugate axis at right angles to the line joining the lights, the position of the bead, when so adjusted in the line joining the two lights, that the luminous bands on either side of it appear of equal width, gives the means of determining their relative power.

There are various dangers on the shores of Britain, more especially at the entrance of the great estuaries of England and also in Ireland, whose position is such as to put them beyond the reach of regular lighthouses. Sand-banks which are too soft to sustain a solid structure, and have too deep water on them to admit of the erection of screw-pile lighthouses, are often the sites for mooring light-vessels, to guide the mariner into the
entrance of some estuary, or enable him to thread his way through the mazes of gats and channels, which, even during the daytime, baffle the mariner, who sees no natural object on the low sandy shores of the neighbouring coast to help him to guess at his true position. The first light-vessel moored on the coast of Great Britain, was that at the Nore in 1734. There are now no fewer than 26 floating lights on the coast of England.

By the kindness of the Elder Brethren of the Corpora-tion of Trinity House of Deptford Strond, I am enabled to give the following brief sketch of the nature and peculiarities of floating lights which was communicated to me by Mr Herbert, the secretary of the Corporation :-
" The annual expense of maintaining a floating light, including the wages and victualling of the crew, who are eleven in number, is, on an average, \(£ 1000\); and the first cost of such a vessel, fitted complete with lantern and lighting apparatus, anchors, cables, \&cc., is nearly £5000. The lanterns are octagonal in form, 5 feet 6 inches in diameter; and, where fixed lights are exhibited, they are fitted with eight Argand lamps, each in the focus of a parabolic reflector of twelve inches diameter; but, in the revolving lights, four lamps and reflectors only are fitted. The greatest depth of water in which any light-vessel belonging to the Corporation of Trinity House of Deptford Strond at present rides, is about 40 fathoms (which is at the station of the Seven Stones between the Scilly Islands and the coast of Cornwall).
"The Corporation's light-vessels are moored with chain-cables of \(1 \frac{1}{\frac{1}{2}}\) inch diameter, and a single mushroom
anchor of 32 cwt ., in which cases the chain-cables are 200 fathoms in length; some of the said vessels are moored to span-ground moorings, consisting of 100 fathoms of chain to each arm, and a mushroom anchor of similar weight at the end of each; a riding cable of 150 fathoms being in such cases attached to the centre ring of the ground chain. The tonnage and general dimensions of the light-vessel are given on the drawing of the lines."
- Still lower in the scale of "signs and marks of the sea," are beacons and buoys, which are used to point out those dangers which, either owing to the difficulty and expense that would attend the placing of more efficient marks to serve by night as well as by day, are necessarily left without lights, or which, from the peculiarity of their position, in passages too intricate for navigation by night, are, in practice, considered to be sufficiently indicated by day-marks alone. Beacons, as being more permanent, are preferred to buoys; but they are generally placed only on rocks or banks which are dry at some period of the tide. On rocks, in exposed situations, beacons are sometimes of squared masonry, secured by numerous joggles; but, in situations difficult of access and in which works of uncompleted masonry could not be safely left during the winter season, an open framework of cast-iron pipes, firmly trussed and braced, and secured to the rock with strong louis-bats, is preferred. The details of this framework are shewn at Plate XIII. A stone beacon of about 40 feet high may be erected for about \(£ 700\), and the iron beacon shewn at Plate XIII. for about £640. In less exposed places, where the bottom is rock,
gravel, or hard sand, a conical form of beacon, composed of cast-iron plates, united with flanges and screws, with rust-joints between them, and partially filled with concrete, is sometimes used. A beacon of that kind can be erected for about \(£ 400\).

Lastly, buoys, which may be regarded as the least efficient kind of mark, and as bearing the same relation to a beacon that a floating-light does to a lighthouse, are used to mark by day dangers which are always coverod even at low water, and also to line out the fairways of channels. They are of three kinds, viz., the Nun-buoy, in the form of a parabolic spindle, generally truncated at one end, so as to carry a mast or frame of cage-work, and loaded at the other end, so as to float in a vertical position; the Can-buoy, which is a conoid floating on its side; and, lastly, the Cask-buoy, which is a short frustum of a spindle truncated at both ends, but almost exclusively used for carrying the warps of vessels riding at moorings. Those buoys are of various sizes and differ in cost. Mast-buoys, from 10 to 15 feet in length, cost from \(£ 23,15 \mathrm{~s}\). to \(£ 48\); and those of the Ribble and the Tay, which are 21 and 24 feet long, cost respectively \(£ 105\) and £79; the Can-buoys are from 5 to 8 feet long, and cost from \(£ 13,13\) s. to \(£ 20\), 5 s . Smaller buoys are also used in narrow estuaries or rivers. Large buoys are often built on kneed frames resembling the timbers of vessels. The Cask-buoy is generally 6 feet long, and costs \(£ 22,15 \mathrm{~s}\). All those buoys are formed of strong oaken barrel-staves, well hooped with iron rings, and shielded with soft timber; and the nozzle-pieces at the small end of the Nun and Can buoys are generally solid
quoins of oak or iron, formed with a raglet or groove to receive the ends of the staves. Much skill, on the part of the cooper, is required in heating and moulding the staves to the required form; and great care must be taken that they be of well-seasoned timber. Buoys are not caulked with oakum, but with dry flags which are closely compressed between the edges of the staves and swell on being wet; and they are carefully proved by steaming them like barrels, to see if they be quite tight. Sheet-iron is sometimes used in making buoys, and they are then sometimes protected with fenders of timber; but they have been found more troublesome for transport and, for most situations, are considered less convenient than those of timber. An attempt has lately been made, under my direction, to construct buoys of gutta percha, stretched on a frame of timber ; but I cannot at present speak confidently of the result.

In the beginning of 1845 , I suggested the idea of rendering beacons and buoys useful during night, by coating them with some phosphorescent substance, or surmounting them with a globe of strong glass filled with such a preparation, whose combustion is very slow, and emits a dull whitish light and little heat. Some experiments were accordingly made ; but no practically useful result has been obtained.

In laying down beacons or buoys, their position is fixed either by the intersection of two lines drawn through two leading objects on the shore (the magnetic bearings of which are given for the sake of easy reference on the spot, in finding out the marks), or by means of the angles contained between lines drawn to
various objects on the shore, which meet at the beacon or buoy from which they are measured by means of a sextant. In the latter case, the angles are always measured around the whole horizon, thus affording a check by the difference of their sum from \(360^{\circ}\). The magnetic bearing of one of those lines is afterwards carefully ascertained, by means of the prismatic compass (if possible from one of the objects on shore, and if not, conversely from the beacon or buoy), so as to afford the means of translating the whole into magnetic bearings for the use of seamen. The buoys are moored by means of chains and iron sinkers, with a sufficient allowance in the length of the chain to permit them to ride easily.

Addendum.-Since the pages ( 55 et seq . Part I.) went to press, in which I have noticed the erection of the tubular iron pillars for the lighthouse on the Bishop Rock off the Scilly Islands, I have learned, with regret, from my friend Mr Walker, the eminent engineer who designed the work, that the whole had disappeared during the gale of the 5th and 6th February last. No account has reached me of the condition in which the rock was found when visited after the storm; and it is, therefore, impossible to form any conjectures as to the mode in which the pillars were destroyed; but such an event naturally increases the doubts which I have ever entertained as to the fitness of such a structure in situations exposed to the force of heavy waves, and strengthens my preference of weight instead of strength as a source of stability.

\section*{APPENDIX.}

\title{
INSTRUCTIONS
}

POB
THE LIGHTKEEPERS
or

\section*{THE N0RTHERN LIGHTH0USES.}
I. The Lamps shall be kept burning bright and clear every night from sunset to sunrise; and in order that the greatest degree of light may be maintained throughout the night, the Wicks must be trimmed every four hours, or oftener if necessary; and the Keeper who has the first watch shall take care to turn the oil valves so as to let the oil flow into the Burner a sufficient time before lighting.
II. The Lightkeepers shall keep a regular and constant Watch in the Light-room throughout the night. The First Watch shall begin at sunset. The Lightkeepers are to take the Watches alternately, in such manner that he who has the first Watch one night shall have the second Watch next night. The length or duration of the Watch shall not, in ordinary cases, exceed four hours; but, during the period between the months of October and March, both inclusive, the first Watch shall change at eight o'elock. The Watches shall at all times be so arranged as to have a shift at midnight.
III. At Stations where there is only one Light-room, the daily
duty shall be laid out in two departments, and the Lightkeepers shall change from one department to the other every Saturday night.
IV. First Defartment.-The Lightkeeper who has this department shall, immediately after the morning Watch, polish or otherwise cleanse the Reflectors or Refractors till they are brought to a proper state of brilliancy; he shall also thoroughly cleanse the Lamps, and carefully dust the Chandelier. He shall supply the Burners with cotton, the Lamps with oil, and shall have every thing connected with the Apparatus in a state of readiness for lighting in the evening.
V. Second Defartment.-The Lightkeeper who has this department shall cleanse the glass of the Lantern, lamp-glasses, copper and brass work and utensils, the walls, floors, and balcony of the Light-room, and the apparatus and machinery therewith connected; together with the Tower stairs, passage, doors, and windows, from the Light-room to the Oil cellar.
VI. For the more effectual cleansing of the glass of the Lantern, and management of the Lamps at the time of lighting, both Lightkeepers shall be upon watch throughout the first hour of the first watch every night, during the winter period, between the first day of October and last day of March, when they shall jointly do the duty of the Light-room during that hour. These changes to and from the double watch shall be intimated by the Keepers in the Monthly Returns for October and April. The Lightkeepers must return to the Lighthonse, on all occasions, so as to be in time to attend the Double Watch at Lighting Time during the Winter Months.
VII. At those Stations where there are two Light-rooms, each Lightkeeper shall perform the entire duty of both departments in that Light-room to which he may be especially appointed. But after the first hour of the first Watch, the Lightkeeper who
has charge of this watch shall perform the whole duty of trimming and attending the Lights of both Light-rooms till the expiry of his watch; and, in like manner, his successor on the watch shall perform the whole duty of both Light-rooms during his watch.
VIII. The Lightkeeper on duty shall on no pretence whatever, during his watch, leave the Light-room and balcony, or the passage leading from one Light-room to another, at stations where there are two Lights. Bells are provided at each Light-room to enable the Lightkeeper on duty to summon the absent Lightkeeper; and if at any time the Lightkeeper on duty shall think the presence or assistance of the Lightkeeper not on duty is necessary, he shall call him by ringing his bell, which should be immediately answered by the return-signal, and the Keeper so called, should repair to the Light-room without delay. In like manner, when the watches come to be changed, the bell shall be rung to call the Lightkeeper next in turn. After which the Lightkeeper on duty shall, at his peril, remain on guard till he is relieved by the Lightkeeper in person who has the next wateh.
IX., Should the bell of the Lightkeeper, whose turn it is to mount guard, happen to be in an unserviceable state, the other house-bell shall be used, and some of the inmates of that house shall call the Lightkeeper not on duty, so as by all means to avoid leaving the Light-room without a constant watch during the night.
X. The Principal Lightkeeper is held responsible for the safety and good order of the Stores, Utensils, and Apparatus of what kind soever; and for every thing being put to its proper use, and kept in its proper place. He shall take care that none of the stores or materials are wasted, and shall observe the strictest economy, and the most careful management, yet so as to maintain in every respect the best possible light.
XI. The Principal Lightkeeper shall daily serve out the allow-
ance of Oil and other Stores for the use of the Light-room. The Oil is to be measured by the Assistant, at the sight of the Principal Lightkeeper. The Lightkeepers are on no account to leave the Turning-keys attached to the Cranes of the Oil Cisterns, after drawing Oil, but shall remove and deposit them on the Tray beside the Oil measures.
XII. The Lightkeeper shall keep a daily Journal of the quantity of Oil expended, the routine of their duty, and the state of the Weather, embodying any other remarks that may occur. These shall be written in the Journal-Books, to be kept at each Station for the purpose, at the periods of the day when they occur, as they must on no account be trusted to memory. On the first day of each month they shall make up and transmit to the Engineer a return, which shall be an accurate copy of the Journal for the preceding month.
XIII. The Lightkeepers are also required to take notice of any Shipwreck which shall happen within the district of the Light:house, and to enter an account thereof, according to the prescribed form, in a Book furnished to each Station for this purpose; and in such account they shall state whether the Light was seen by any one on board the shipwrecked Vessel and recognized by them, and how long it was seen before the Vessel struck. A copy of this entry shall form the Shipwrecked Return, to be forthwith forwarded to the Engineer. When any application is made to the Lightkeepers for information as to shipwrecks in the neighbourhood of the Station, or as to the state of the weather, or the management and arrangement of the Lighthouses, they are to direct the applicants to communicate with the Engineer or Secretary of the Board in Edinburgh.
XIV. A Book, containing a note of the Vessels passing each Lighthouse, shall be kept; and an annual Schedule, shewing the number of Vessels in each month, shall be sent to the Engineer in the month of January.
XV. The Monthly and Shipwreck Returns are to be written by the Assistant, and the accompanying letters by the Principal Lightkeeper. The whole shall be carefully compared, and the additions of the various columns tested by both Lightkeepers, who shall also sign the same as correct, according to the printed form, and the Principal Keeper shall despatch by post to the Engineer as soon as possible. The accounts presented at the Lighthouses are to be carefully examined and checked by both Lightkeepers, before being sent to the Engineer.
XVI. For each Station there shail be at least one person resident near the Lighthouse, who shall be fully instructed in the whole practice of the Light-room duty, and shall come under an agreement to be always ready to take the place of either Keeper, in case of sickness or other emergency. This person shall be called the " Occasional Keeper;" and he shall be duly instructed in the duty, and qualified for this trust by a residence in the Lighthouse, and course of training in the whole of the duty during not less than three weeks for the Reflecting, and four weeks for the Refracting Lights. For the purpose of keeping up the practical knowledge of the "Occasional Keeper," he shall be annually called in by the Principal Lightkeeper to do duty for a fortnight in the month of January ; and the same shall be stated in the Monthly Letter. At inaccessible Stations, where there are two "Occasional Keepers," they shall each serve a fortnight in January, and shall, as much as possible, be called in alternately to do duty.
XVII. The Principal Lightkeeper is held responsible for the regularity of the Watches throughout the night, for the cleanliness and good order of the Reflecting or Refracting Apparatus, Machinery, and Utensils, and for the due performance of the whole duty of the Light-room or Light-rooms, as the case may be, whether performed by him personally, or by the Assistant.
XVIII. The Principal Lightkeeper is also held responsible for
the good order and condition of the Household Furniture belonging to the Lighthouse Board, as well in his own as in the Assistant's house. This duty extends also to the cleanliness of the several apartments, passages, stairs, roofs, water-cisterns, storerooms, workshops, privies, ash-pits of the dwelling-houses, offices, court, and immediate access to the Lighthouse.
XIX. The Lightkeepers shall endeavour to keep in good order and repair the Dykes enclosing the Lighthouse grounds, the Landing-places and Roads leading from thence to the Lighthouse, and the Drains therewith connected; together with all other things placed under their charge.
XX. When stores of any kind are to be landed for the use of the Lighthouse, the Lightkeepers shall attend and give their assistance. The Principal Lightkeeper must, upon these occasions, satisfy himself, as far as possible, of the quantity and condition of the Stores received, which must be duly entered in the Store-book and Monthly Return-book.
XXI. The Lightkeepers are to make a Report of the quality of the Stores, in the Monthly Return for March annually, or earlier should circumstances render this necessary; and this Report must proceed upon special trial of the several Cisterns of Oil and the other stores in detail, both at the time of receiving them and after the experience of the winter months.
XXII. At all Stations where Peat Fuel is in use, there must be such a quantity of Peats provided, that the Stock of the former year shall be a sufficient supply to the end of the current year.
XXIII. Should the supply of any of the Lighthouse Stores at any time appear to the Principal Lightkeeper to be getting short, so as thereby to endanger the regular appearance of the Light, he shall immediately intimate the same to the Engineer; and he must
be guided by prudence in reducing the stated number of Burners until a supply be received.
XXIV. The Lightkeepers are prohibited from carrying on any trade or business whatever. They are also prohibited from having any boarders or lodgers in their dwelling-houses, and from keeping dogs at the Lighthouse establishments.
XXV. The Lightkeepers are also directed to take care that no smuggled goods are harboured or concealed in any way in or about the Lighthouse premises or grounds.
XXVI. The Lightkeepers have permission to go from home to draw their salaries, and also to attend church. The Assistant Lightkeeper, on all occasions of leave of absence, must consult the Principal Lightkeeper as to the proper time for such leave, and obtain his consent; in like manner, the Principal Lightkeeper shall duly intimate his intention of going from home to the Assistant Lightkeeper;-it being expressly ordered that only one Lightkeeper shall be absent from the Lighthouse at one and the same time.
XXVII. While the Principal Lightkeeper is absent, or is incapacitated for duty by sickness, the full charge of the Lightroom duty and of the premises shall devolve upon the Assistant, who shall in that case have access to the keys of the Lightroom stores, and be held responsible in all respects as the Principal Lightkeeper; and in case of the incapacity of either Lightkeeper, the assistance of the Occasional Lightkeeper shall be immediately called in, and notice of the same given to the Engineer. Notice to be taken of any such occurrences in the Monthly Return, or by special letter to the Engineer, should circumstances render this necessary.
XXVIII. The Lightkeepers are required to be sober and industrious, cleanly in their persons and linens, and orderly in their
families. They must conduct themselves with civility to strangers, by shewing the premises, at such hours as do not interfere with the proper duties of their office; it being expressly understood, that strangers shall not be admitted into the Light-room after sunset. But no money or other gratuity shall be taken from strangers on any pretence whatever.
XXIX. Frequent intimations having been made in the newspapers as to Pleasure Excursions to Islands on which Lighthouse Establishments are placed, and representations having been made of the danger of injury to the Lighting Apparatus from parties crowding the Light-room, it is hereby directed that, when more than twelve persons visit a Station, the Light-room and Lighting Apparatus shall not be open to inspection; and it is further directed, that not more than three strangers shall have access to any Light-room at one and the same time. The Lightkeepers must not on any pretext admit persons in a state of intoxication. The Lighthouse Establishments are not to be open to the public on Sunday. No stranger visiting a Light-room shall be permitted to handle any part of the Apparatus, or make Drawings, or to take any Dimensions, unless he shall produce a written authority for so doing from an Officer of the Board.
XXX. The Lightkeepers are to appear in their Uniform-dress when any of the Commissioners or Principal Officers visit a Station, and also on Sunday ;-on which day, at noon, the weather permitting, the Lighthouse flag shall be hoisted by the Assistant Lightkeeper, or in his absence by the Principal Lightkeeper, when it shall remain displayed until sunset.
XXXI. In collecting the Accounts due to tradesmen and others near the Station at the Half-yearly Terms, the Principal Keeper must send the exact Address of the parties to whom money is to be paid, so that an order for payment may reach them directly from the Office. There are also issued two Papers, one of which is called " List of Accounts delivered," and the other " Account of

Sums claimed." The first Paper must contain a list of the small Sums mentioned below, and his own customary claims. In the second Paper the Keeper will enter his own claims in detail, the amount of which will also be entered in the first; and he will, at the same time, give a copy of the Form to the Assistant for the entry of his claims. The Principal Keeper will send direct to the Engineer all other Accounts incurred for Work done at the Lighthouse, as soon as the Work is completed, without waiting till the end of the half-year. The Half-yearly List will thus merely include any small Accounts, such as those for Washing, Carriage of Letters and Parcels, Rents of Seats in Church, Shoeing the Horse, and other small customary charges from the Keepers.
XXXII. In the event of any neglect occurring in the performance of any part of the duties required from a Lightkceper, the offending party shall, jointly with the other Lightkeeper or Lightkeepers at the Station, send immediate notice of the circumstance to the Engineer; and, in the event of one party refusing or neglecting to concur in giving this intimation, the others (whether Principals or Assistants) shall proceed to give the notice in their own names.
XXXIII. The breach of any of the foregoing Rules and Instructions shall subject the Lightkeepers to dismissal, or to such other punishment as the nature of the offence may require.
XXXIV. It is recommended that the Principal Light-keeper or other Principal Officer at the respective Lighthouses for the time being, shall, every Sunday, perform the service pointed out for the inmates, by reading a portion of the Scriptures, and any other religious book furnished by the Board, and the prayer composed for their use by the Rev. Dr Brunton, one of the Ministers of Edinburgh, or other Prayers in any work furnished by the Board. For this purpose, the Principal Lightkeeper shall invite the families to assemble at noon in the Visiting Officers's room.
XXXV. The Lightkeepers are to observe that the above general Regulations are without prejudice to any more special Instructions which may be made applicable to any particular Lighthouse, or to such orders as may from time to time be issued by the Engineer.
XXXVI. As the Commissioners have no power to make provisions for the widows and children of persons in their service, they directed, by Minute of 5th February 1839, that in lien thereof, the sum of \(£ 3\) per annum be added to each Lightkeeper's Salary, to be retained, and applied towards effecting an Insurance on his Life, under the following Regulations, which are here repeated for the information and guidance of the Lightkeepers :-
1. That the sum retained for Insurance shall in no event be payable to or assignable by any Lightkeeper, or attachable by his creditors.
2. That the addition made to the Lightkeeper's Salaries shall be employed in paying an annnal premiam on a Policy of Insurance on their respective lives, for such a sum as can be obtained for such premium, in reference to the age of the party insured; the policy for such insurance to be taken in name of the Commissioners, and the sums contained in them to be payable to the Commissioners after the death of the Lightkeeper.
3. That it shall be competent to every Lightkeeper, at any time during his life, by any writing under his hand, either in the form of a letter to the Secretary, or otherwise, to direct in what manner he would wish the sum arising from his Policy of Insurance, at his death, to be applied for the benefit of his wife and family; but it is to be understood that the Board consider it inexpedient that in any case they should retain the sum coming into their hands for behoof of such widow or family; and that in the event of no written directions being left, the Board will apply such sums for beboof of the widow or family, one or both, at their discretion, in such way as they may think most beneficial, without having regard to the manner in which such sums might fall to be divided at common law.
4. That in the event of no such directions for behoof of the widow and family being left by the Lightkeeper, the Policies, and sums arising from them, shall be held by the Board exclusive of the creditors or other assignees of the deceased Lightkeeper, the object being to provide a fund to the widow and family, where such are left.
5. That in the event of any Lightkeeper being at any time dismissed from, or leaving, the service of the Board, he is to have no claim on the Policy, nor for the annual premiums paid, nor for its value,-the same, in that case, being entirely at the disposal of the Board; with power to them, nevertheless, if they shall see cause, to give up the Policy to the party dismissed or leaving the service, or to his faraily, or to give its value, or otherwise to dispose of it.
6. That in the event of any Lightkeeper dying, and leaving no wife and family, the Board will still apply the sums coming into their hands in terms of any written directions he may leave; but failing such, and, on his dying intestate, the Board will leave the Insurance Office to settle with his next of kin according to the usual forms of law.
7. That from the Salary of every Lightkeeper hereafter entering the service, a sum of \(£ 3\) shall be retained annually to effect an Insurance on his life, subject to the above conditions.
XXXVII. These Instructions are to be read in the Light-room by the Principal Lightkeeper, in the hearing of his Assistant, on the term days, before drawing his salary; and notice of such reading taken in the Monthly Returns.
(Signed) ALAN STEVENSON,
Engineer.

Office of the Board of Northery Liohthouses,
Edinburge, 12th December 1849.

The Commissioners having considered the preceding Rules and Instructions, approve of the same; direct them to be substituted for those now in use; appoint them to be signed by the Engineer, and copies of them and of this Minute to be issued to the present Lightkeepers; direct a copy to be delivered in future to each Lightkeeper at the time of his appointment, that they may understand that they are placed under the department and superintendence of the Engineer, who is held responsible for the strict observance of the Rules and Instructions, and for their general good conduct; that the Engineer has power, in case of neglect or disobedience, instantly to suspend and remove any of the Lightkeepers, and to report the case to the Commissioners, by whom it will be considered, and the offending party subjected to dismissal, or such other punishment as the offence may merit. In case of a punishment less than dismissal, that circumstance, as well as the general conduct of the Lightkeeper, will always be taken into consideration when any application may be made for the Superannuated Allowance.

> Extracted from the Minutes by

ALEX. CUNINGHAM, Secy.



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26, 27. Plan, elevation and cross section of a lift lock, Sandy and Beaver canal.
28. Gate, front view ; front view of falling gate, mitre sill, section, \&e.

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34, 35. Plan of a wooden lock of 8 feet lift, zeveral sections.
36, 37. Plan of Rivanna aquednct, in elevations and sections, horizontal section at surface of water, plan of pier abutment, and wing-walls, \&c.
88. Farm Bridge, James river and Kanawha canal; elevation, plan, longitudinal and cross section.
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